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Specific Energy
&
Hydraulic Jump

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Reference

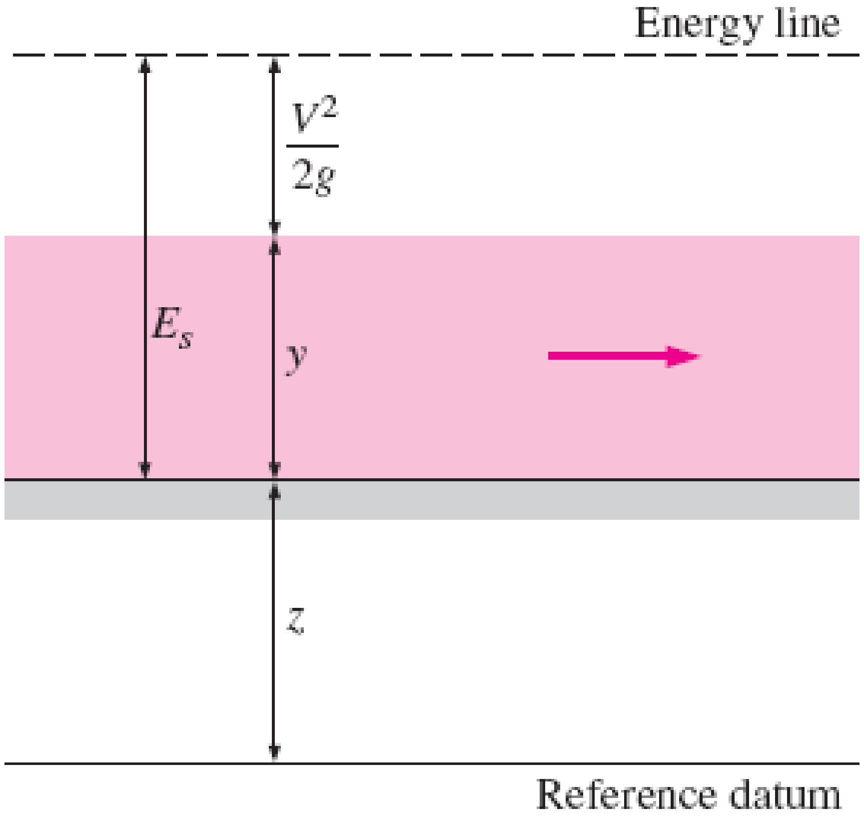
Fluid Mechanics: Fundamentals and Applications
2nd EDITION IN SI UNITS
Yunus A. Cengel, John M. Cimbala
McGraw-Hill, 2010

Chapter 13
OPEN-CHANNEL FLOW

Lecture slides by

Mehmet Kanoglu

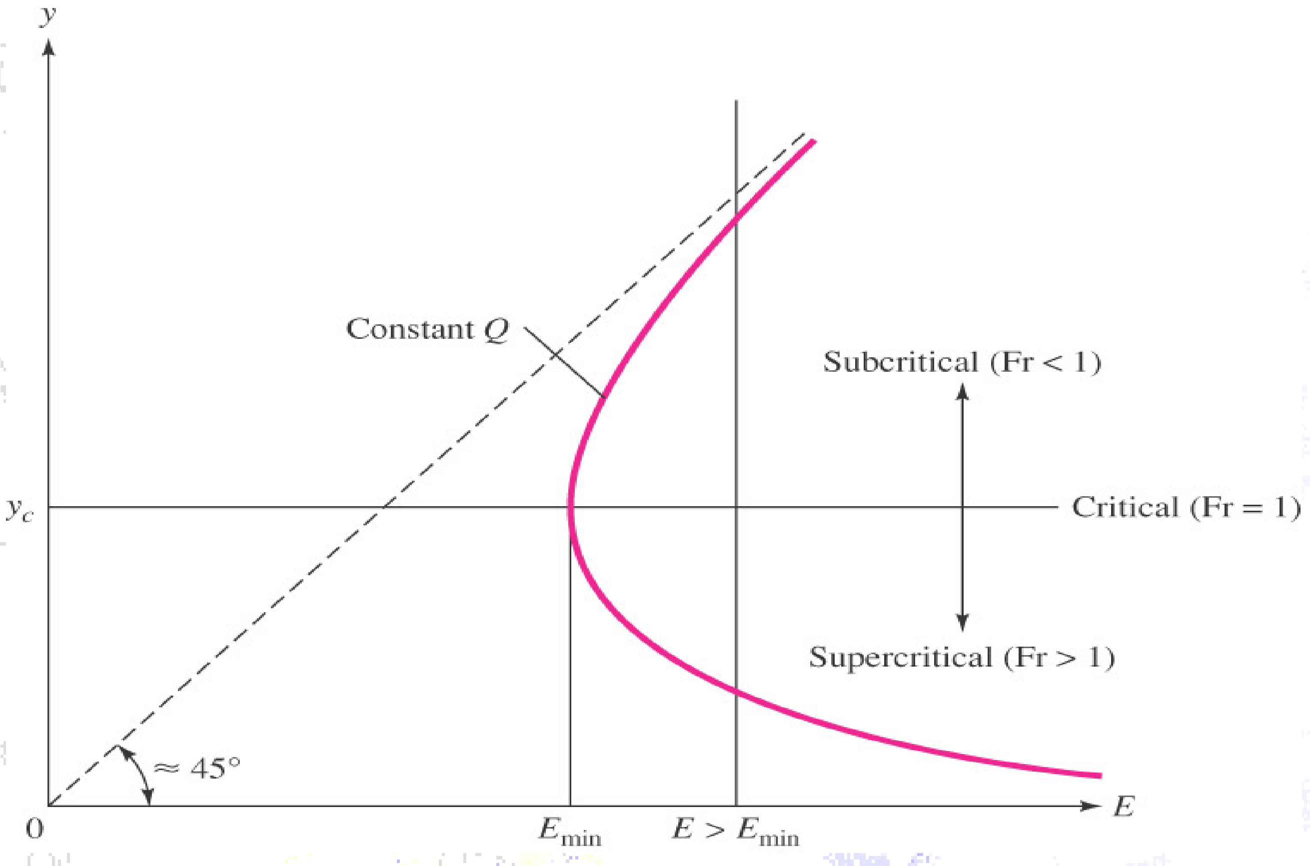
SPECIFIC ENERGY



$$E_s = y + \frac{V^2}{2g}$$

The specific energy Es of a liquid in an open channel is the total mechanical energy (expressed as a head) relative to the bottom of the channel.

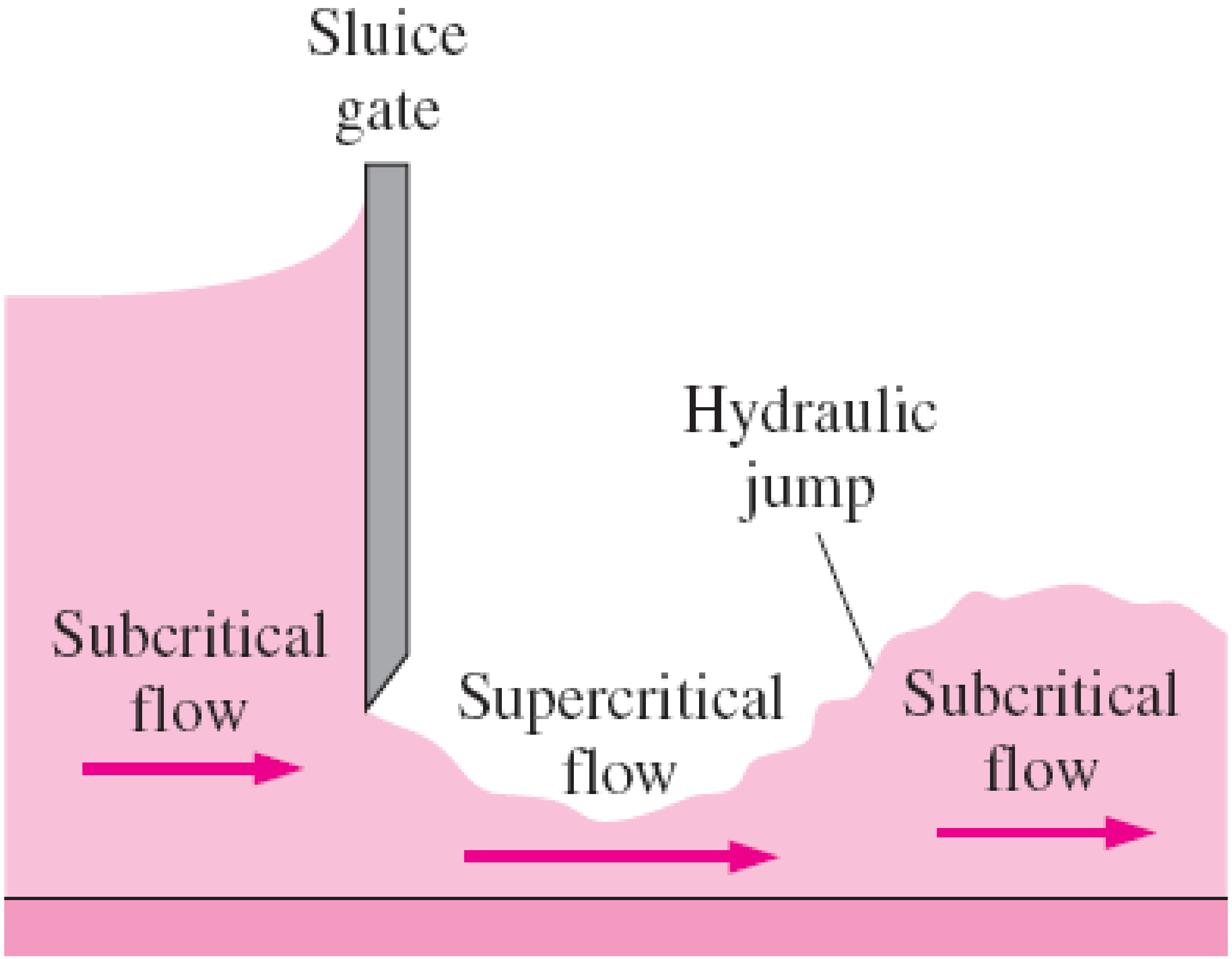
SPECIFIC ENERGY



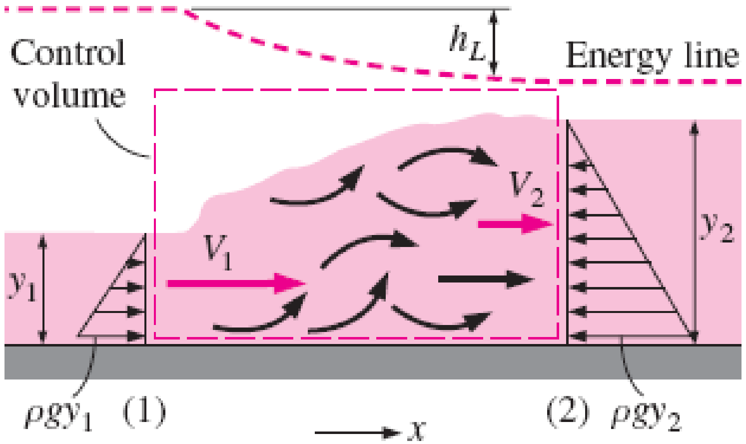
The specific energy reaches a minimum value *Es, min* at some intermediate point, called the critical point, characterized by the critical depth *yc* and critical velocity *Vc*.

The minimum specific energy is also called the critical energy.

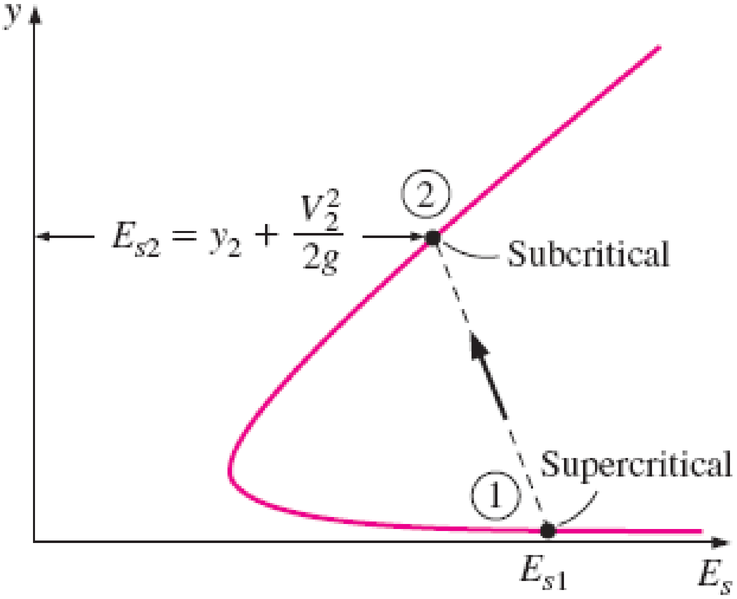
Hydraulic jump



Schematic and flow depth-specific energy diagram for a hydraulic jump (specific energy decreases).



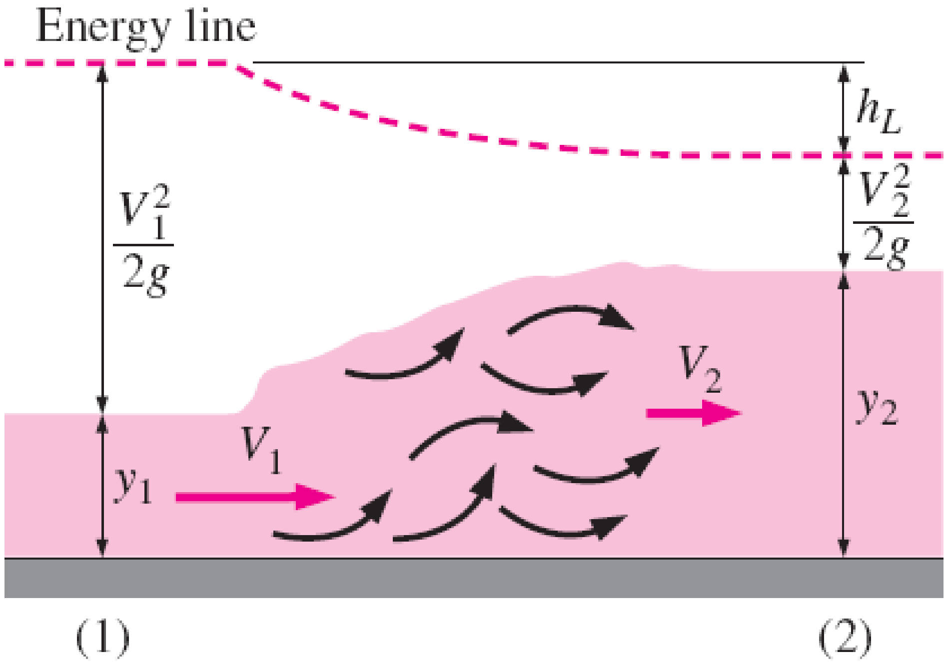
Depth ratio: $\frac{y_2}{y_1} = 0.5\left(-1 + \sqrt{1 + 8\mathrm{Fr}_1^2}\right)$



$$y_1 + \frac{V_1^2}{2g} = y_2 + \frac{V_2^2}{2g} + h_L$$

$$h_L = y_1 - y_2 + \frac{V_1^2 - V_2^2}{2g} = y_1 - y_2 + \frac{y_1\mathrm{Fr}_1^2}{2}\left(1 - \frac{y_1^2}{y_2^2}\right)$$

$$\text{Energy dissipation ratio} = \frac{h_L}{E_{s1}} = \frac{h_L}{y_1 + V_1^2/2g} = \frac{h_L}{y_1(1 + \mathrm{Fr}_1^2/2)}$$



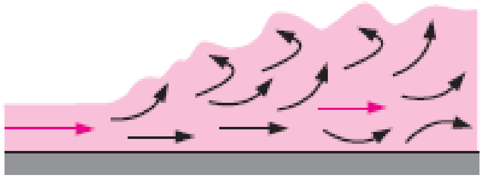
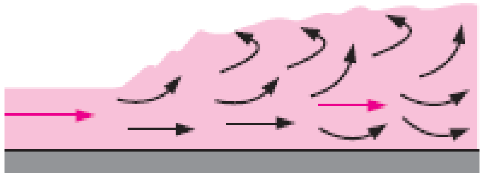
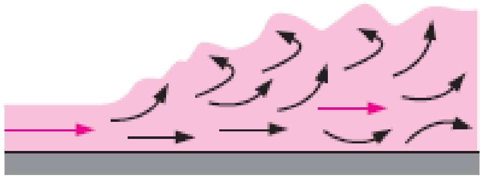
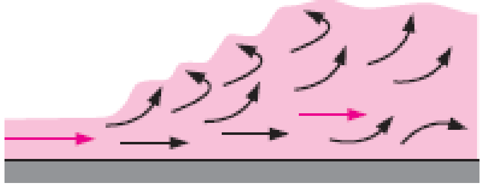
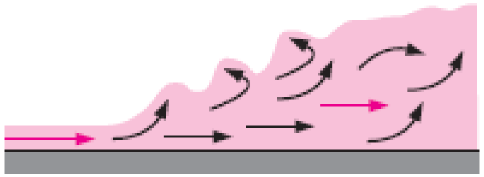
$$\text{Dissipation ratio} = \frac{h_L}{E_{s1}} = \frac{h_L}{y_1 + V_1^2/2g}$$

The fraction of energy dissipation ranges from just a few percent for weak hydraulic jumps ($\mathrm{Fr}_1 < 2$) to 85 percent for strong jumps ($\mathrm{Fr}_1 > 9$).

The energy dissipation ratio represents the fraction of mechanical energy dissipated during a hydraulic jump.

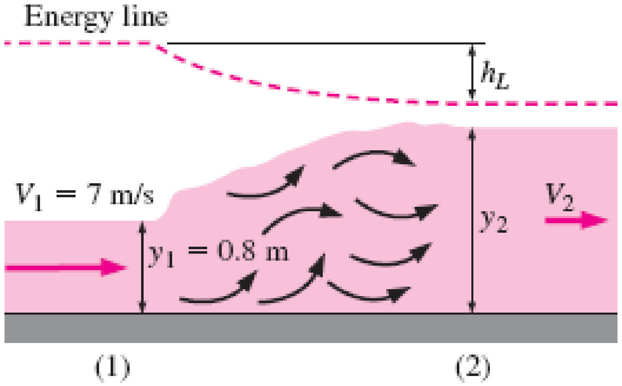
Classification of hydraulic jumps

Source: U.S. Bureau of Reclamation (1955).

Upstream Fr_1	Depth Ratio y_2/y_1	Fraction of Energy Dissipation	Description	Surface Profile
<1	1	0	*Impossible jump*. Would violate the second law of thermodynamics.	
1–1.7	1–2	<5%	*Undular jump* (or *standing wave*). Small rise in surface level. Low energy dissipation. Surface rollers develop near Fr = 1.7.	
1.7–2.5	2–3.1	5–15%	*Weak jump*. Surface rising smoothly, with small rollers. Low energy dissipation.	
2.5–4.5	3.1–5.9	15–45%	*Oscillating jump*. Pulsations caused by jets entering at the bottom generate large waves that can travel for miles and damage earth banks. Should be avoided in the design of stilling basins.	
4.5–9	5.9–12	45–70%	*Steady jump*. Stable, well-balanced, and insensitive to downstream conditions. Intense eddy motion and high level of energy dissipation within the jump. Recommended range for design.	
>9	>12	70–85%	*Strong jump*. Rough and intermittent. Very effective energy dissipation, but may be uneconomical compared to other designs because of the larger water heights involved.	

EXAMPLE 13–8 Hydraulic Jump

Water discharging into a 10-m-wide rectangular horizontal channel from a sluice gate is observed to have undergone a hydraulic jump. The flow depth and velocity before the jump are 0.8 m and 7 m/s, respectively. Determine (*a*) the flow depth and the Froude number after the jump, (*b*) the head loss and the energy dissipation ratio, and (*c*) the wasted power production potential due to the hydraulic jump (Fig. 13–41).



SOLUTION Water at a specified depth and velocity undergoes a hydraulic jump in a horizontal channel. The depth and Froude number after the jump, the head loss and the dissipation ratio, and the wasted power potential are to be determined.

Assumptions **1** The flow is steady or quasi-steady. **2** The channel is sufficiently wide so that the end effects are negligible.

Properties The density of water is 1000 kg/m^3.

Analysis (*a*) The Froude number before the hydraulic jump is

$$\mathrm{Fr}_1 = \frac{V_1}{\sqrt{gy_1}} = \frac{7\ \mathrm{m/s}}{\sqrt{(9.81\ \mathrm{m/s^2})(0.8\ \mathrm{m})}} = 2.50$$

which is greater than 1. Therefore, the flow is indeed supercritical before the jump. The flow depth, velocity, and Froude number after the jump are

$$y_2 = 0.5y_1\left(-1 + \sqrt{1 + 8\mathrm{Fr}_1^2}\right) = 0.5(0.8\ \mathrm{m})\left(-1 + \sqrt{1 + 8 \times 2.50^2}\right) = \mathbf{2.46\ m}$$

$$V_2 = \frac{y_1}{y_2}V_1 = \frac{0.8\ \mathrm{m}}{2.46\ \mathrm{m}}(7\ \mathrm{m/s}) = 2.28\ \mathrm{m/s}$$

$$\mathrm{Fr}_2 = \frac{V_2}{\sqrt{gy_2}} = \frac{2.28\ \mathrm{m/s}}{\sqrt{(9.81\ \mathrm{m/s^2})(2.46\ \mathrm{m})}} = \mathbf{0.464}$$

Note that the flow depth triples and the Froude number reduces to about one-fifth after the jump.

(*b*) The head loss is determined from the energy equation to be

$$h_L = y_1 - y_2 + \frac{V_1^2 - V_2^2}{2g} = (0.8\text{ m}) - (2.46\text{ m}) + \frac{(7\text{ m/s})^2 - (2.28\text{ m/s})^2}{2(9.81\text{ m/s}^2)}$$

$$= \mathbf{0.572\text{ m}}$$

The specific energy of water before the jump and the dissipation ratio are

$$E_{s1} = y_1 + \frac{V_1^2}{2g} = (0.8\text{ m}) + \frac{(7\text{ m/s})^2}{2(9.81\text{ m/s}^2)} = 3.30\text{ m}$$

$$\text{Dissipation ratio} = \frac{h_L}{E_{s1}} = \frac{0.572\text{ m}}{3.30\text{ m}} = \mathbf{0.173}$$

Therefore, 17.3 percent of the available head (or mechanical energy) of the liquid is wasted (converted to thermal energy) as a result of frictional effects during this hydraulic jump.

(*c*) The mass flow rate of water is

$$\dot{m} = \rho\dot{V} = \rho b y_1 V_1 = (1000\text{ kg/m}^3)(0.8\text{ m})(10\text{ m})(7\text{ m/s}) = 56{,}000\text{ kg/s}$$

Then the power dissipation corresponding to a head loss of 0.572 m becomes

$$\dot{E}_{\text{dissipated}} = \dot{m}gh_L = (56{,}000\text{ kg/s})(9.81\text{ m/s}^2)(0.572\text{ m})\left(\frac{1\text{ N}}{1\text{ kg}\cdot\text{m/s}^2}\right)$$

$$= 314{,}000\text{ N}\cdot\text{m/s} = \mathbf{314\text{ kW}}$$

Discussion The results show that the hydraulic jump is a highly dissipative process, wasting 314 kW of power production potential in this case. That is, if the water were routed to a hydraulic turbine instead of being released from the sluice gate, up to 314 kW of power could be generated. But this potential is converted to useless thermal energy instead of useful power, causing a water temperature rise of

$$\Delta T = \frac{\dot{E}_{\text{dissipated}}}{\dot{m}c_p} = \frac{314\text{ kJ/s}}{(56{,}000\text{ kg/s})(4.18\text{ kJ/kg}\cdot{}^\circ\text{C})} = 0.0013^\circ\text{C}$$

Note that a 314-kW resistance heater would cause the same temperature rise for water flowing at a rate of 56,000 kg/s.

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Thank you