Open Channel Flow Resistance[1]

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Abstract: In 1965, Rouse critically reviewed hydraulic resistance in open channels on the basis of fluid mechanics. He pointed out the effects of cross-sectional shape, boundary nonuniformity, and flow unsteadiness, in addition to viscosity and wall roughness that are commonly considered. This paper extends that study by discussing the differences between momentum and energy resistances, between point, cross-sectional and reach resistance coefficients, as well as compound/composite channel resistance. Certain resistance phenomena can be explained with the inner and outer laws of boundary layer theory. The issue of linear-separation approach versus nonlinear approach to alluvial channel resistances also is discussed. This review indicates the need for extensive further research on the subject.

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Introduction

The distinct honor and opportunity to present this Hunter Rouse Award Lecture has special meaning to the writer. It was Dr. Rouse, in the process of preparing for his classic article on the subject (Rouse 1965), who challenged the writer and got him interested in hydraulic resistance beyond steady uniform flow in prismatic channels. Rouse (1965) classified flow resistance into four components: (1) surface or skin friction, (2) form resistance or drag, (3) wave resistance from free surface distortion, and (4) resistance associated with local acceleration or flow unsteadiness. By using the Weisbach resistance coefficient *f*, he expressed the resistance as the following dimensionless symbolic function:

$$f = F(\mathsf{R}, \mathbf{K}, \eta, \mathbf{N}, \mathsf{F}, \mathbf{U}) \tag{1}$$

in which R = Reynolds number; $\mathbf{K}$ = relative roughness, usually expressed as k_s/R, where k_s is the equivalent wall surface roughness and R is hydraulic radius of the flow; η = cross-sectional geometric shape; $\mathbf{N}$ = nonuniformity of the channel in both profile and plan; F = Froude number; $\mathbf{U}$ = degree of flow unsteadiness; and F represents a function. The symbolic relationship of Eq. (1) can also be applied to the Manning resistance coefficient n in the form of $n/k_s^{1/6}$, or to a flow resistance slope S. Leopold et al. (1964) divided the resistance into those due to skin friction, internal distortion, and spills.

Rouse (1965) aptly showed that what is now commonly called the Moody diagram is a special case of Eq. (1) for steady uniform flow in straight constant diameter rigid pipes, considering only two of the six independent parameters in Eq. (1), namely, the flow Reynolds number R and the relative roughness k_s/R for Nikuradse-type dense random surface roughness. Obviously, the six independent parameters in Eq. (1), as well as the four resistance components (surface, form, wave, and unsteadiness), interact in a nonlinear manner such that any linear separation and combination is artificial.

The most frequently used formulas relating open-channel flow velocity, V, to resistance coefficient are

$$V = \frac{K_n}{n} R^{2/3} S^{1/2} \quad \text{(Manning)} \tag{2}$$

$$V = \sqrt{\frac{8g}{f}} \sqrt{RS} \quad \text{(Darcy–Weisbach)} \tag{3}$$

$$V = C\sqrt{RS} \quad \text{(Chezy)} \tag{4}$$

in which n, f, and C are the Manning, Weisbach, and Chezy resistance coefficients, respectively; R = hydraulic radius, S = slope; g = gravitational acceleration; and $K_n = 1\ \mathrm{m}^{1/2}/\mathrm{s}$ for V and R in SI units, 1.486 $\mathrm{ft}^{1/3}$-$\mathrm{m}^{1/6}/\mathrm{s}$ for English units, and $\sqrt{g}$ for dimensionally homogeneous Manning formula (Yen 1992). From Eqs. (2)–(4), the resistance coefficients can be related as

$$\sqrt{\frac{f}{8}} = \frac{n}{R^{1/6}} \frac{\sqrt{g}}{K_n} = \frac{\sqrt{g}}{C} = \frac{\sqrt{gRS}}{V} \tag{5}$$

Thus, knowing the value of one resistance coefficient, the corresponding values of the other resistance coefficients can be computed.

Over the years there have been numerous investigators who have made important contributions to open-channel flow resistance. It is impossible to include even a small portion of these previous accomplishments in this short presentation. The purpose of this presentation is following Rouse's idea of Eq. (1) to further examine the resistance coefficients from the perspectives of fluid mechanics and channel boundary, based mostly on the writer's and a few related experiences. Hopefully, this presentation will promote new interests and advances in channel resistance.

Wall Surface Resistance and Boundary Layer Theory

Among the four component types of channel flow resistance classified by Rouse (1965), the wall surface or skin friction resistance

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(hereafter simply referred to as wall resistance) always exists and can readily be linked to the boundary layer theory in fluid mechanics. It has long been suspected that flow resistance is related to the velocity distribution. Stokes (1845) suggested the internal tangential shear stress τ_{ij} proportional to the molecular dynamic viscosity μ and the velocity gradient, i.e.,

$$\tau_{ij}=\mu\left(\frac{\partial u_i}{\partial x_j}+\frac{\partial u_j}{\partial x_i}\right) \tag{6}$$

in which u_i is the local point velocity component in the x_i direction. It is worthwhile to note that, earlier, Saint-Venant (1843) proposed a similar but more general expression,

$$\tau_{ij}=\varepsilon\left(\frac{\partial u_i}{\partial x_j}+\frac{\partial u_j}{\partial x_i}\right) \tag{7}$$

which is applicable to laminar flow when $\epsilon=\mu$ and to turbulent flow when ϵ is an apparent viscosity coefficient which includes the molecular dynamic viscosity and the turbulent or eddy viscosity due to turbulence averaging.

At the wall boundary of the channel, the local point shear, τ_0, is

$$\tau_0=\mu\left.\frac{d\mathbf{v}}{dy}\right|_{y=0} \tag{8}$$

where y is in the direction normal to the wall and $\mathbf{v}$=velocity vector. The distribution of $\mathbf{v}$, in general, is affected by the geometry of the channel. Consider the special case of steady flow over a large smooth flat wall (or two-dimensional wide channel without Taylor vortex cells in the lateral direction) with $\mathbf{v}=u$ along the longitudinal direction x only. In accordance with the boundary layer theory, the distribution of u along the wall-normal y direction is adequately described by two universal laws, namely, the inner law or law of the wall where the viscous effect dominates, and the outer law or velocity defect law (Rouse 1959; Hinze 1975; Schlichting 1979), i.e.,

$$\frac{u}{u_*}=F(y^*,k^*) \tag{9}$$

$$\frac{U-u}{u_*}=F\left(\frac{y}{\delta},H_s\right) \tag{10}$$

in which $u_*=\sqrt{\tau_0/\rho}$ is a kinematic shear measure usually called shear velocity because of its dimension, where ρ=density of the fluid and τ_0=local point wall shear stress; $y^*=u^*y/\nu$ where $\nu=\mu/\rho$ is the kinematic viscosity of the fluid; U=free stream velocity at the far end of the outer law; δ=boundary layer thickness; and H_s, often called a shape factor, is a nondimensional parameter associated with the pressure gradient and Reynolds number, and it is usually expressed as the ratio between the displacement and momentum thicknesses of the boundary layer.

The regions of inner and outer laws are not mutually exclusive. There is an overlapping region between the lower limit of the outer law y_1 and the upper limit of the inner law y_2 as shown in Fig. 1. In this overlapping region, the equations of the inner law and outer law [Eqs. (9) and (10)] both apply. It can be proved (Rouse 1959, pp. 348–350) that a logarithmic function,

$$\frac{u}{u_*}=c_1\log y^*+c_2 \tag{11}$$

satisfies Eqs. (9) and (10) simultaneously. In Eq. (11), c_1 and c_2 are constants for a given channel. There is likely a family of other

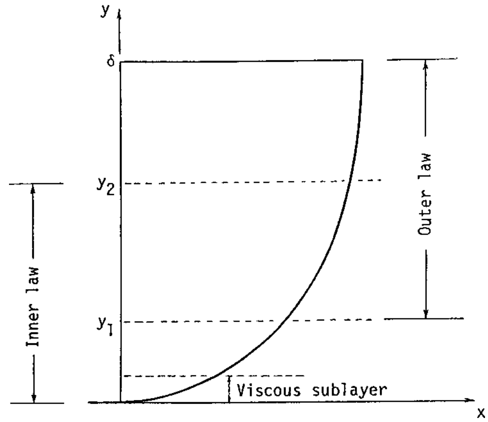


Fig. 1. Regions of boundary layer inner and outer laws (from Yen 1991)

functions that can also satisfy simultaneous the inner and outer law equations, e.g., the often used power law distribution (Chen 1991)

$$\frac{u}{u_*}=c_3(y^*)^m \tag{12}$$

where c_3 is a constant. The exponent m usually ranges between $\frac{1}{4}$ and $\frac{1}{12}$ for different boundaries. Another possibility is an exponential type distribution (Chiu et al. 1993).

For the lower limit of the overlapping region, Schlichting (1979) suggested a value of $y_1^*=70$. Hinze (1975) indicated that y_1^* may be as low as 30. Rouse (1959) also recommended $y_1^*=30$.

In the inner law region, for a very thin layer immediately adjacent to the wall, the viscous effect dominates and the Reynolds stresses, $\rho\overline{u_i'u_j'}$, are negligible and the fluid motion is essentially instantaneously laminar. Within this viscous sublayer, the local shear stress is constant, i.e., $\tau=\mu u/y$, or, together with Eq. (8), yields

$$\frac{u}{u_*}=y^* \quad \text{for } y^*\leqslant 4 \tag{13}$$

Direct measurement of wall shear stress is rather difficult. Successful attempts are rare (O'Loughlin 1965; Petryk and Shen 1971). Local point wall shear is usually computed from u_* which is determined through measured velocity distribution fitted to Eqs. (9), (10), (11), or (12), or by using instruments such as the Preston tube that are based on velocity distribution. The region of the inner law below the overlapping region (Fig. 1) is usually thin and difficult to measure the velocity, especially when the wall roughness is large. Therefore, usually the velocity measurements are made in the overlapping region for which the logarithm distribution [Eq. (11)] applies. One should note that the shape factor H_s in Eq. (10) indicates that the coefficients c_1 and c_2 in Eq. (11), and hence, the local wall shear, depend on the channel geometry.

Much of the current idea on open-channel resistance is derived and extended from resistance of steady uniform flow in straight axisymmetric (and hence, mathematically speaking two dimensional) circular rigid pipes. Besides a half circle, the closest open-channel counterpart of a circular pipe is the two-dimensional (2D) wide channel. With the shape factor held constant for 2D wide channels or circular pipes, Eqs. (9) and (10) imply that, by referring to Eq. (1), the resistance to a steady uniform flow is only a

Table 1. Values of Constants for Colebrook–White-Type Formula for Steady Uniform Flow in Open Channels with Rigid Impervious Boundary (from Yen 1991)

Channel geometry	Reference	K_1	K_2	K_3	Remarks
Full circular pipe	Colebrook (1939)	2.0	14.83	2.52	
Wide channel	Keulegan (1938)	2.03	11.09	3.41	
Wide channel	Rouse (1946, p. 214)	2.03	10.95	1.70	
Wide channel	Thijsse (1949)	2.03	12.2	3.033	
Wide channel	Sayre and Albertson (1961)	2.14	8.888	7.17	
Wide channel	Henderson (1966)	2.0	12.0	2.5	
Wide channel	Graf (1971, p. 305)	2.0	12.9	2.77	
Wide channel	Reinius (1961)	2.0	12.4	3.4	
Rectangular	Reinius (1961)	2.0	14.4	2.9	Width/depth=4
Rectangular	Reinius (1961)	2.0	14.8	2.8	Width/depth=2
Rectangular	Zegzhda (1938)	2.0	11.55	0	Dense sand

function of the Reynolds number R and relative roughness $\mathbf{K} = k_s/R$, provided that the Froude number is not high and its effect is negligible. Hence

$$f, \frac{n}{R^{1/6}}, \frac{C}{\sqrt{g}}, = F\left(\mathsf{R}, \frac{k_s}{R}\right) \tag{14}$$

This is the basis of the Moody (1944) diagram. For steady uniform laminar flow with Reynolds number $\mathsf{R} = VR/\nu < 500$ (V = cross sectional average velocity and R = hydraulic radius), the resistance coefficient in Darcy–Weisbach form is

$$f = K_L/\mathsf{R} \tag{15}$$

where $K_L = 24$ for 2D wide channels and 16 for circular pipes. For $700 < \mathsf{R} < 25{,}000$ Blasius' (1913) formula for smooth pipes is

$$f = 0.224/\mathsf{R}^{0.25} \tag{16}$$

which is often used as an approximation for wide channels. For $\mathsf{R} > 25{,}000$, the Colebrook–White type formula is often used

$$\frac{1}{\sqrt{f}} = -K_1 \log\left(\frac{k_s}{K_2 R} + \frac{K_3}{4\mathsf{R}\sqrt{f}}\right) \tag{17}$$

Some suggested values of the coefficients K_1, K_2, and K_3 are listed in Table 1. Taking the 2D wide channel as reference, the values of K_L [Eq. (15)] and K_3 [Eq. (17)] decrease with decreasing channel width to depth ratio, while K_2 increases. Based on Eqs. (15)–(17), a Moody-type diagram for wide open channels is sketched in Fig. 2. More experimental data and theoretical analyses are needed for precise locations of the curves in Fig. 2, particularly for other geometry shapes.

The Colebrook–White formula [Eq. (17)] is implicit in f. Suggestions have been made to overcome this drawback. The most successful is perhaps the following (Barr 1972; Churchill 1973; Barr 1977) for full-flow pipes

$$f = \frac{1}{4}\left[-\log\left(\frac{k_s}{14.8R}\right) + \frac{5.76}{(4\mathsf{R})^{0.9}}\right]^{-2} \tag{18}$$

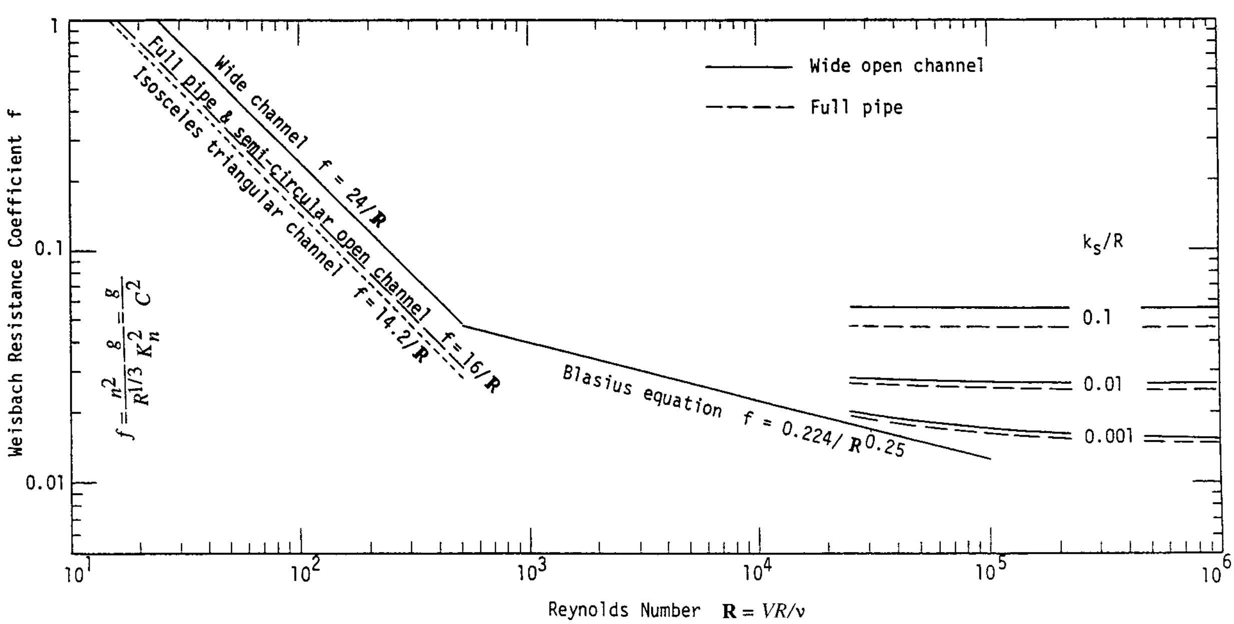


Fig. 2. Moody type diagram for open channels with impervious rigid boundary

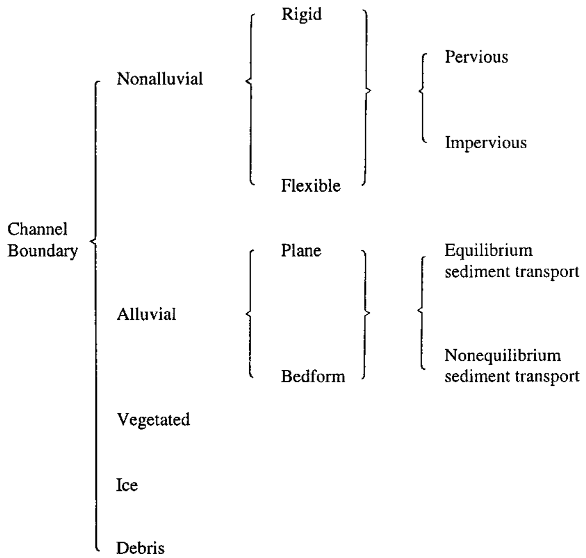


Fig. 3. Channel boundary classification

(Barr used the exponent 0.89 instead of 0.9 and constant 5.2 instead of 5.76). Yen (1991) suggested a provisional corresponding explicit formula for 2D wide open channels with $\mathsf{R} > 30{,}000$ and $k_s/R < 0.05$

$$f = \frac{1}{4}\left[-\log\left(\frac{k_s}{12R} + \frac{1.95}{\mathsf{R}^{0.9}}\right)\right]^{-2} \tag{19}$$

In this computer age, there have been considerable interests to represent the Moody diagram in a single equation. One possibility is to incorporate probability into the resistance functions [Eqs. (15)–(17)] of the three-flow regions with the occurrence probabilities P_1, P_2, and $P_3(=1-P_1-P_2)$, respectively (Yen 1991). Thus

$$f = P_1F_1(\mathsf{R}) + P_2F_2(\mathsf{R}) + (1-P_1-P_2)F_3\left(\mathsf{R}, \frac{k_s}{R}\right) \tag{20}$$

As indicated in Eq. (8), the shear on the boundary surface depends on the velocity gradient at the boundary. The Moody diagram is for rigid impervious boundaries, a special case of the general boundary surface types listed in Fig. 3. Permeability of the wall surface with lateral in or out flow, wall mobility or flexibility, ice or debris on the free surface, and vegetation or obstacles in the channel all modify the velocity distribution and, hence, the wall shear stress.

Hydraulically, it is important to differentiate between the flow resistance characteristics of an alluvial (sediment laden) boundary and those of a nonalluvial channel boundary. Fig. 3 depicts a classification of channel boundaries for steady flows. A nonalluvial boundary has a nondetachable bed surface or bed roughness. The alluvial boundary is composed of sediment particles which are movable with the flowing water, and water can flow through the voids between the particles in the bed.

Momentum versus Energy for Resistance Coefficients

In the preceding section, wall surface resistance is considered in terms of boundary shear, i.e., from the viewpoint of force and impulse momentum relationship. Flow resistance can also be reviewed from the energy concept in terms of the energy lost as the fluid moves. In open channels, the flow resistance is often expressed in terms of a slope as indicated in Eq. (5). The momentum resistance slope along the x_i direction for a channel cross section, S_{mi}, can be expressed as (Yen 1973)

$$S_{mi} = \frac{-1}{\gamma A}\int_\sigma [\tau_{ij}]N_j d\sigma \tag{21}$$

in which γ = specific weight of the fluid; A = flow cross sectional area normal to x_i direction and bounded by the boundary σ; N_j = directional normal of σ along x_j direction; and the shear stresses acting on σ is

$$\tau_{ij} = \mu\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right) - \rho\overline{u_i'u_j'} \tag{22}$$

where u_i' is the turbulence fluctuation with respect to the local mean (over turbulence) velocity component u_i.

Conversely, the energy slope, or more precisely, the gradient of the dissipated mean-motion (over turbulence and viscosity) energy, S_e, is

$$S_e = \frac{1}{\gamma AV}\int_A \tau_{ij}\frac{\partial u_i}{\partial x_j} dA \tag{23}$$

in which all the symbols have been defined previously. In Eqs. (21)–(23), u and τ are mean quantities over turbulence; for simplicity a bar is not given on top of these variables as commonly done in fluid mechanics because the fluctuation quantities such as $\overline{u_i'u_j'}$ are only occasionally dealt herewith.

In general, for a given flow, S_{mi} and S_e are different not only numerically but also, more importantly, in concept.

1. S_{mi} is a vector quantity with specific directions following the momentum concept, whereas S_e is a scalar quantity following the energy concept.
2. S_{mi} results from the external forces acting on the boundary of the control volume or on the perimeter of the cross section. It is not directly a function of the flow inside the cross section or control volume. Conversely, S_e represents the energy dissipated directly through viscosity into heat and the apparent loss of energy into turbulence through the transport and decay of the eddies that cannot be recovered by the mean motion. In other words, it represents the work done by the flow against the internal forces generated from molecular viscosity and eddy viscosity overcoming the flow velocity gradient. Therefore, for a rigid boundary the energy losses occur inside the control volume or the cross section. In fact, for a fixed wall where there is no velocity, the work done is zero along the solid wall.
3. Thus, mathematically, the momentum resistance slope S_{mi} involves a line integral [for a cross section, Eq. (21)] or a surface integral (for a control volume) of the ith-direction component of the external forces acting on the perimeter; whereas the energy resistance slope S_e involves an area integral [cross section, Eq. (23)] or a volume integral (for a reach) of the work done by internal forces inside the area or volume.

It is appropriate to clarify here the difference between a roughness factor and a resistance coefficient. A roughness factor is a geometry measure reflecting the actual or effective unevenness of the boundary. A resistance coefficient is a measure reflecting the dynamic behavior in terms of momentum or energy, of the boundary in resisting the flow of the fluid.

Following the development of fluid mechanics, especially with the influence of the Prandtl–Karman Gottingen school, most engineering fields consider the resistance coefficient as a momentum phenomenon. Conversely, in civil and agricultural engineering, following a long history of measurements in streams and canals, flow resistances have been treated more or less in an energy manner. This difference causes confusion not only in concepts but also in proper determination of coefficient values and analyses. Rouse (1961, 1962) is a pioneer in basic examination of energy losses by investigating the work-done terms inside the integral of Eq. (23).

Like the momentum and energy resistance slopes, S_m and S_e from Eq. (5), the Weisbach, Manning, or Chezy resistance coefficients can also be expressed following the momentum and energy concepts, respectively, as f_m, f_e and n_m, n_e. Only for steady uniform flow in a straight prismatic channel with a rigid impervious wall and without lateral flow does $S_m = S_e = S_0$ numerically, where S_0 is the channel slope. In general, these slopes are numerically, as well as conceptually, different. They are also different from the following slopes that often appear in open channel analyses (Yen 1973):

1. The friction slope, S_f, which accounts for only the shear resistance from the wetted perimeter;
2. Water surface slope with respect to channel longitudinal direction, $S_w = \partial h/\partial x$, or to a horizontal plane, $S_w' = (\partial h/\partial x) - S_0$, where h is the depth of flow;
3. $S_H = -\partial H/\partial x$, the slope of conventionally used approximately total head

$$H = H_p + z_b + \frac{V^2}{2g} \tag{24}$$

where H_p = cross-sectional average piezometric head with respect to the channel bottom, and z_b is the bed elevation above the reference horizontal datum;
4. The slope of actual total head

$$H_B = \left(\frac{BV_iV_i}{2g}\right) + Kh\cos\theta + z_b \tag{25}$$

where B and K = correction factors for velocity and pressure distributions, respectively; θ = angle between the channel bed along x direction and the horizontal datum; and V_iV_i denotes the square of the mean velocity vector;
5. The piezometric-head slope with respect to the channel bed, $\partial H_p/\partial x$;
6. The hydraulic gradient, $(\partial H_p/\partial x) - S_0$ which is the gradient of H_p with respect to a horizontal reference plane.

Yen et al. (1972) demonstrated that for shallow flow under lateral inflow such as rainfall (at a rate i), f_e, f_m, and f_H can be considerably different in magnitude, as is illustrated by Fig. 4. In Fig. 4, h and V = depth and velocity of the flow; d = equivalent diameter of raindrops; and i = intensity of rainfall as lateral inflow with velocity U when joining the main flow. The higher the value of i/U, the more energy input from the lateral flow. It can be seen in Fig. 4 that the energy resistance coefficient f_e is consistently higher than the corresponding momentum coefficient f_m. Based on the data in Fig. 4, as well as other data and an analysis by Shen and Li (1973), Yen suggested the following formula instead of Eq. (15) for the momentum equation of laminar sheet flow under rainfall with an intensity i (Yen and Akan 1999),

$$f_m = \frac{8g}{K_n^2}\left(\frac{n_m}{R^{1/6}}\right)^2 = \left[24 + 660\left(\frac{i}{\sqrt[3]{g\nu}}\right)^{0.4}\right]\frac{1}{\mathsf{R}} \tag{26}$$

In general, even for steady flow, accurate information on the momentum and energy resistance coefficients is almost nonexistent, except for uniform flow in pipes or 2D channels—the case of the Moody diagram.

Manning *n*, Chezy *C* versus Weisbach *f*

Equation (5) can be applied to momentum or energy resistance coefficients for a point, a cross section or a reach. It also shows that because the Weisbach, Manning, and Chezy coefficients can be related, there is no clear theoretical advantage of one coefficient over the others. Therefore, a comparison of the three coefficients and formulas from a practical viewpoint may be useful.

Historically, the Weisbach f has the advantage of being directly related to the development of fluid mechanics by the scientists in Gottingen and other places, and hence, it is sometimes misquoted as being a theoretical coefficient. The Chezy formula is the simplest in form and has the longest history. Manning's n has the advantage of being nearly a constant almost independent of flow depth, Reynolds number or k_s/R for fully developed turbulent flow over a rigid rough surface. Those interested in the error of using n may refer to Yen (1991, 1992). The most authoritative source for the values of f is the Moody diagram. The most common sources for n is the table in Chow (1959) and the picture book of Barnes (1967). There is no generally recognized table or figure for Chezy's C.

In fluid mechanics, f is usually associated with the shear-momentum concept. It is almost only hydraulic engineers, and rarely engineers in other disciplines, who would consider f an energy loss coefficient. Generally, f is regarded as a point value related to the velocity distribution, although some hydraulic engineers extend it to cross section or reach values and consider it as an energy loss coefficient as well.

On the other hand, the determination of the resistance coefficient values by Manning using field data followed the head loss energy concept applied to channel reaches, although in Manning's (1891) article, channel wall resistance was mentioned. The calculated values are channel reach energy loss coefficient values and in hydraulic engineering n has been used as such. Thus, it appears appropriate to refer Weisbach's f for point resistance while Manning's n for cross sectional and reach resistance coefficients. Field experiences in the past may suggest n to be a simpler coefficient to accommodate the effects of other parameters in Eq. (1) in addition to the Reynolds number and relative roughness.

The relationship between n and f given in Eq. (5) allows calculation of the equivalent roughness k_s used in the Moody diagram corresponding to given n values (Yen 1991). Example values of k_s converted from the n values in Chow (1959) for concrete channels are listed in Table 2. A more complete table of k_s for channels of other types of wall surfaces can be found in Yen (1991).

From a fluid mechanics perspective, one drawback of the Manning formula is that it is dimensionally nonhomogeneous. As an improvement Yen (1991, 1992), Dooge (1991), as well as, Mostafa and McDermid (1971) suggested modifying the Manning equation to the following form:

$$V = M\left(\frac{R}{k_s}\right)^{1/6}\sqrt{gRS} \tag{27}$$

in which $M = (K_n/\sqrt{g})(k_s^{1/6}/n)$. To minimize alternation from the Manning equation but retaining the physical meaning of gravity in the flow, Yen (1991) recommended using the Manning formula in the following dimensionally homogeneous form:

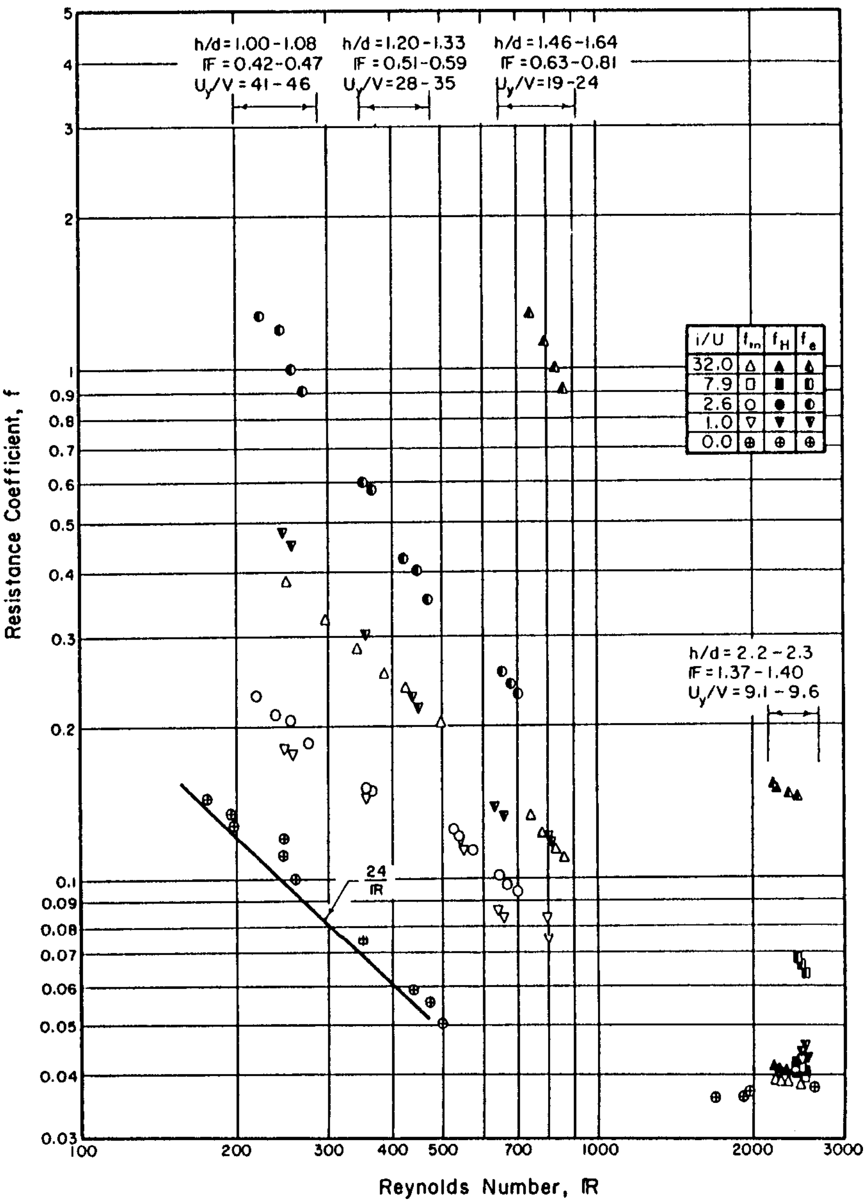


Fig. 4. Difference between momentum and energy resistance coefficients for steady sheet flow with lateral inflow of rain intensity i (from Yen et al. 1972)

Table 2. Values of Equivalent Roughness k_s for Concrete Channels

Type of surface	Minimum mm	Minimum 10^{-3} ft	Normal mm	Normal 10^{-3} ft	Maximum mm	Maximum 10^{-3} ft
1. Trowel finish	0.5	1.7	1.5	4.5	3.3	10
2. Float finish	1.5	4.5	3.3	10	5.0	15
3. Finished, with gravel on bottom	3.3	10	7.0	23	18	60
4. Unfinished	2.0	7	7.0	23	18	60
5. Shortcrete, good section	5.0	15	14	45	43	140
6. Shortcrete, wavy section	10	30	33	100	70	230
7. On good excavated rock	7.0	23	18	60		
8. On irregular excavated rock	33	100	112	360		

$$V=\frac{\sqrt{g}}{n_g}R^{2/3}S^{1/2} \quad (28)$$

where $n_g=(\sqrt{g}/K_n)n$.

Point, Cross Section, and Reach Resistance Coefficients

Open-channel computations or measurements are often made reach by reach or cross section to cross section, and the resistance coefficient should be determined accordingly. For steady uniform flow in straight axisymmetric full-flow pipes or 2D wide open channels (without Taylor vortex instability), the point wall shear, and hence, the value of resistance coefficient are the same as those of cross section or reach. Otherwise, the geometric shape parameter η and sometimes the nonuniformity parameter $\mathbf{N}$ in Eq. (1) will become influential factors. Fig. 5 shows the ranges of space scale for point, cross section, and reach of common open

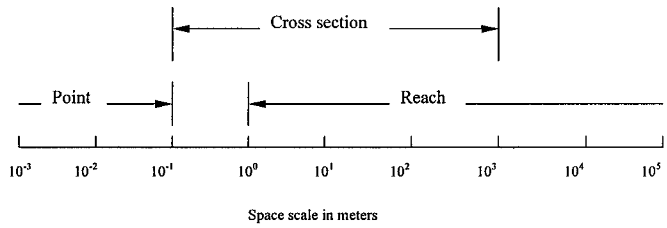


Fig. 5. Space scales for point, cross section, and reach resistance

channels. The ''point'' scale extends beyond a point because of the uneven roughness elements or bed forms that actually exist on the boundary surface and the point shear velocity (and hence, wall shear) is determined indirectly from the velocity distribution [Eqs. (11) or (12)] which is a reflection of a small area rather than truly a point of the wall boundary surface. In applying and extending fluid mechanics theories to open-channel resistance coefficients, realization of the scale factor of point, section, and reach, as well as the differences between energy and momentum coefficients, is of paramount importance.

From Eq. (5) for a steady uniform flow in a prismatic channel, the average shear stress is $\overline{\tau_0}=\gamma RS$. Thus, $u_*=\sqrt{\overline{\tau_0}/\rho}=\sqrt{gRS}$. In general, for nonuniform flows or for nonprismatic channels, $\overline{\tau_0}\neq\gamma RS$ because of the pressure gradient and longitudinal component of the pressure acting on the channel boundary. Nonetheless, the last term in Eq. (5) is often written as u_*/V with u_* defined and determined in different ways as shown in Fig. 6.

As to the average velocity V in Eq. (5), following the inner and outer laws (Fig. 1) the average velocity along the normal from the channel wall, V_h, is

$$V_h=\frac{1}{h}\left[\int_0^y u\,dy+\int_{y'}^h u\,dy\right] \tag{29}$$

Strictly speaking, h in Eq. (29) should end at the upper limit of the outer law, often the point of maximum velocity if it is not the water surface along the wall normal. However, in calculation using experimental data, the error involved in extending the limit of the second integral from location of maximum velocity to the water surface is usually small. Conversely, the error in taking the depth along vertical instead of wall normal may be significant.

Likewise, for a channel reach, the average velocity, V_R, can be defined differently. Three definitions of V_R are listed in Fig. 6(c). The hydraulic radius R of the reach can also be computed in two different ways as listed in Fig. 6(c). Following the shear-momentum concept, for the six definitions of u_* and three definitions of V listed in Fig. 6(c), there are 18 different definitions of u_*/V as shown. Thus, there are 18 different ways to compute the average Manning n or Weisbach f for the reach, in addition to computing them as the average of the local n or f, indicated as alternatives at the bottom of Fig. 6(c).

Similarly, there are four different ways to determine the cross sectional n or f [Fig. 6(b)], and three different ways to determine the point n or f [Fig. 6(a)]. Among them, those expressed in terms of $\sqrt{gRS}$ are applicable to energy resistance coefficient in addition to the momentum resistance coefficient. Three of them, u_{*R2}/V_{R2} for reach resistance, u_{*a}/V_a for cross sectional resistance, and u_{*3}/V_h for point resistance are in agreement with the definitions of the Manning, Darcy–Weisbach, and Chezy equations shown in Eq. (5). With so many different definitions and ways of computation for the resistance coefficients, no wonder there are confusions and inconsistency in values for the resistance coefficients.

Because accurate wall shear determination is relatively difficult and time consuming, reach resistance is usually determined through comparing the total heads at two (end) or more cross sections of the channel reach. Most of the Manning n values determined from field measurements use this method (e.g., Barnes 1967), corresponding to the energy approach for the case of u_{*R2}/V_{R2} in Fig. 6(c). From the practical viewpoint, accuracy can be enhanced and the analysis can be simpler if the two end sections are similar in geometry and flow condition. In other words, ideally the reach should be selected such that the flow is ''reachwise uniform,'' with the flow at the two end sections identical although the flow in between inside the reach can be nonuniform because of the nonprismatic channel geometry.

Resistance of Composite or Compound Channels

A composite channel is a channel whose wall roughness changes along the wetted perimeter of the cross section. Consequently, in general, the local point wall shear and resistance also vary along the wetted perimeter. In other words, the resistant coefficient is affected by $\mathbf{K}=k_s/R$ in Eq. (1) as this parameter varies along the wall. A compound channel, strictly speaking, is a channel whose cross section consists of subsections of different defined geometric shapes put together. For the rare case of homogeneous wall roughness, a primary factor affecting the resistance coefficient is the geometric parameter η in Eq. (1). However, in field situations compound channels are generally also composite, e.g., the case of main channel with floodplains. Thus, it is conceivable that the value of the resistance coefficient for the cross section or reach changes with the flow depth.

Physically, the composite/compound roughness on the wall as well as the shape of the channel modifies the velocity distribution across the cross section, and hence alters the resistance coefficient. A detailed examination on the effects of varying wall roughness and cross sectional geometry would require a 2D or three-dimensional (3D) analysis of the flow. For a one-dimensional (1D) analysis of steady flow in a straight channel of rigid impervious boundary without sediment, Eq. (1) with refinement on $\mathbf{K}$ and $\mathbf{N}$ can be written as

$$f,\frac{n}{R^{1/6}} \text{ or } S=F\left(\mathsf{R},\mathsf{F},S_w,S_0,\eta,\frac{k_s}{R},G_c\right) \tag{30}$$

in which S_0=channel bed slope; S_w=water surface slope; and G_c=a nondimensional representation of the lateral variation of the wall roughness along the wetted perimeter of the flow cross section. G_c can be expressed, e.g., as k_i/k_s where k_s is regarded as the cross sectional mean roughness value and k_i is the local roughness of the ith part of the wetted perimeter.

Traditionally, instead of using Eq. (30), the compound/composite roughness resistance coefficient of a cross section is conventionally expressed in the Manning n form, with the cross sectional value, n_c, being a weighted sum of the local resistance factor, n_i, i.e.,

$$n_c=\int_P w_i n_i\,dp \tag{31a}$$

in which w_i is the weighing function and P is the wetted perimeter. In computation, usually a finite discretization approach is used by dividing the cross section into a number of subcross sections of area A_i, wetted perimeter P_i, and hydraulic radius R_i. The weighing factor w_i is a function of these parameters, and

$$n_c=\sum_i w_i n_i \tag{31b}$$

Definition of Wall Shear τ_o	Definition of Wall Shear Velocity u_*	$\frac{u_*}{V}$
Measured τ_o	$u_{*1} = \sqrt{\tau_o/\rho}$	u_{*1}/V_h
Computed $\tau_o = \rho u_*^2$	u_{*2} Computed from velocity distribution: Log distribution between y_1 and h: Smooth $\frac{V_h}{u_*} = c_1 \log \frac{hu_*}{\upsilon} + c_2$ Rough $\frac{V_h}{u_*} = c_1 \log \frac{h}{k_s} + c_3$ Power law distribtution $\frac{V_h}{u_*} = c_4 \left(\frac{hu_*}{\upsilon}\right)^{m+1}$ or $\frac{V_h}{u_*} = c_5 \left(\frac{h}{k_s}\right)^{m+1}$	u_{*2}/V_h
$\tau_o = \gamma hS$	$u_{*3} = \sqrt{ghS}$	u_{*3}/V_h

(*a*)

Definition of Wall Shear Stress τ	Definition of Wall Shear Velocity u_*	$\frac{u_*}{V}$
$\tau_w = \frac{1}{P}\int_p \tau_o dP$	$u_{*w} = \sqrt{\tau_w/\rho}$	u_{*w}/V_a
	$\bar{u}_{*o} = \frac{1}{P}\int_P \sqrt{\frac{\tau_o}{\rho}}\, dP$	$\bar{u}_{*o}/V_a$
$\tau_a = \gamma RS$	$u_{*a} = \sqrt{gRS}$	u_{*a}/V_a

$$V_a = \frac{Q}{A} = \frac{1}{A}\int_P \int_o^h u\, dydP$$

(*b*) Note alternative: $n_a = \frac{1}{P}\int_p n_p dP$, $f_a = \frac{1}{P}\int_P f_p dP$

Definition of Reach Wall Shear τ_R	Hydraulic Radius	Definition of Reach Shear Velocity u_{*R}	Reach Mean Velocity V_R	$\left(\frac{u_*}{V}\right)_{reach}$
$\tau_{RS1} = \gamma \bar{R}_1 \bar{S}$	$\bar{R}_1 = \frac{1}{L}\int_o^L R dx$	$u_{*R1} = \sqrt{g \bar{\bar{R}}_1 \bar{\bar{S}}}$	V_{R1} V_{R2} V_{R3}	u_{*R1}/V_{R1} u_{*R1}/V_{R2} u_{*R1}/V_{R3}
$\tau_{RS2} = \gamma \bar{R}_2 \bar{S}$	$\bar{R}_2 = \frac{\frac{1}{L}\int_0^L A dx}{\frac{1}{L}\int_0^L P dx} = \frac{\bar{A}}{\bar{P}}$	$u_{*R2} = \sqrt{g \bar{R}_2 \bar{S}}$	V_{R1} V_{R2} V_{R3}	u_{*R2}/V_{R1} u_{*R2}/V_{R2} u_{*R2}/V_{R3}
τ_o of point		$u_{*R3} = \frac{1}{L}\int_o^L \frac{1}{P}\int_P \sqrt{\frac{\tau_o}{\rho}}\, dPdx = \frac{1}{L}\int_o^L \frac{1}{P}\int_P \bar{u}_{*o}\, dPdx$	V_{R1} V_{R2} V_{R3}	u_{*R3}/V_{R1} u_{*R3}/V_{R2} u_{*R3}/V_{R3}
$\tau_{RW} = \frac{1}{L}\int_o^L \frac{1}{P}\int_P \tau_o dPdx = \frac{1}{L}\int_o^L \tau_w dx$		$u_{*R4} = \sqrt{\tau_{RW}/\rho}$	V_{R1} V_{R2} V_{R3}	u_{*R4}/V_{R1} u_{*R4}/V_{R2} u_{*R4}/V_{R3}
τ_w of cross section		$u_{*R5} = \frac{1}{L}\int_o^L \sqrt{\frac{\tau_w}{\rho}}\, dx = \frac{1}{L}\int_o^L u_{*w}\, dx$	V_{R1} V_{R2} V_{R3}	u_{*R5}/V_{R1} u_{*R5}/V_{R2} u_{*R5}/V_{R3}
τ_a of cross section	Cross sectional R	$u_{*R6} = \frac{1}{L}\int_o^L \sqrt{gRS}\, dx = \frac{1}{L}\int_o^L u_{*a}\, dx$	V_{R1} V_{R2} V_{R3}	u_{*R6}/V_{R1} u_{*R6}/V_{R2} u_{*R6}/V_{R3}

$$V_{R1} = \frac{1}{L}\int_o^L \frac{Q}{A} dx, \quad V_{R2} = \frac{Q}{\bar{\bar{A}}} = Q\Big/\frac{1}{L}\int_o^L A dx, \quad V_{R3} = \frac{1}{L}\int_o^L \frac{1}{P}\int_P \frac{1}{h}\int_o^h u dydPdx$$

(*c*) *Note alternatives*: $n_R = \frac{1}{L}\int_o^L n_a dx = \frac{1}{L}\int_o^L \frac{1}{P}\int_P n_p dP$, $f_R = \frac{1}{L}\int_o^L f_a dx = \frac{1}{L}\int_o^L \frac{1}{P}\int_P f_p dP$

Fig. 6. Expressions of u_*/V for computation of Manning *n* or Weisbach *f*: (a) point resistance; (b) cross section resistance; and (c) reach resistance

Table 3. Equations for Compound or Composite Channel Resistance Coefficient

Eqs.	n_c	Assumptions: Concept	Assumptions: Equation	Reference
A	$=\frac{\Sigma n_i A_i}{A}$	Sum of component n weighted by area ratio; or Total shear velociety is weighted sum of subarea shear velociety	$\sqrt{gRS}=\Sigma\left(\frac{P_i}{P}\sqrt{gR_iS_i}\right)$ $(V_i/V)=(R_i/R)^{7/6}$	U.S. Army Corps of Engineers Los Angeles District Method, see Cox (1973)
B	$=\sqrt{\Sigma n_i^2\frac{A_i}{A}}$	Total resistance force is equal to sum of subarea resistance forces; or, n_i weighted by $\sqrt{A_i}$	$P\gamma RS=\Sigma P_i\gamma R_iS_i$ $(V_i/V)=(R_i/R)^{2/3}$	
C	$=\frac{A}{\Sigma(A_i/n_i)}$	Total discharge is sum of subarea discharges	$Q=VA=\Sigma(V_iA_i)=\Sigma Q_i$ $(S_i/S)=(R/R_i)^{4/3}$	
D	$=\left[\frac{\Sigma(n_i^{3/2}A_i)}{A}\right]^{2/3}$	Same as Horton and Einstein's Eq. E but derived erroneously		Colebatch (1941)
E	$=\left[\frac{1}{P}\Sigma(n_i^{3/2}P_i)\right]^{2/3}$	Total cross sectional mean velocity equal to subarea mean velocity	$V=V_i$ $A=\Sigma A_i \qquad S=S_i$	Horton (1933) Einstein (1934)
F	$=\frac{P}{\Sigma(P_i/n_i)}$	Total discharge is sum of subarea discharges	$Q=\Sigma Q_i$ $(S_i/S)=(R/R_i)^{10/3}$	Felkel(1960)
G	$=\left[\frac{1}{P}\Sigma(n_i^2P_i)\right]^{1/2}$	Total resistance force, F, is sum of subarea resistance forces, ΣF_i	$P\gamma RS=\Sigma P_i\gamma R_iS_i$ $(V_i/V)=(R_i/R)^{1/6}$	Pavlovskii (1931)
H	$=\frac{\Sigma(n_iP_i)}{P}$	Total shear velocity is weighted sum of subarea shear velocity; or, Contributing component roughness is linearly proportional to wetted perimeter	$\sqrt{gRS}=\Sigma\left(\frac{P_i}{P}\sqrt{gR_iS_i}\right)$ $(V_i/V)=(R_i/R)^{1/6}$ or $n_cP=\Sigma(n_iP_i)$	Yen (1991)
I	$=\left[\frac{R^{1/3}}{P}\Sigma\frac{n_i^2P_i}{R_i^{1/3}}\right]^{1/2}$	Total resistance force, F, is sum of subarea resistance forces, ΣF_i	$P\gamma RS=\Sigma P_i\gamma R_iS_i$ $(V_i/V)=1$	
J	$=\left[\frac{\Sigma n_i^2P_iR_i^{2/3}}{PR^{2/3}}\right]^{1/2}$	Total resistance force equal to sum of subarea resistance forces	$P\gamma RS=\Sigma P_i\gamma R_iS_i$ $(V_i/V)=(R_i/R)^{1/2}$	
K	$=\frac{PR^{7/6}}{\Sigma\frac{P_i}{n_i}R_i^{7/6}}$	Total discharge is sum of subarea discharges	$Q=VA=\Sigma(V_iA_i)$ $(S_i/S)=(R/R_i)$	
L	$=\frac{PR^{5/3}}{\Sigma\frac{P_iR_i^{5/3}}{n_i}}$	Total discharge is sum of subarea discharges	$Q=VA=\Sigma(V_iA_i)$ $(S_i/S)=1$ $R=A/P$	Lotter (1993)
M	$=\frac{\Sigma P_iR_i^{5/3}}{\Sigma\frac{P_iR_i^{5/3}}{n_i}}$	Same as Eq. L with modified definition of R	$Q=VA=\Sigma(V_iA_i)$ $(S_i/S)=1$ R from $\frac{PR^{5/3}}{\Sigma P_iR_i^{5/3}}=\frac{AR^{2/3}}{\Sigma A_iR_i^{2/3}}=1$	Ida (1960) Engelund (1964)
N	$=\frac{\Sigma(n_iP_i/R_i^{1/6})}{P/R^{1/6}}$	Total shear velocity, $\sqrt{gRS}$ is weighted sum of subarea shear velocity	$\sqrt{gRS}=\Sigma\left(\frac{P_i}{P}\sqrt{gR_iS_i}\right)$ $(V_i/V)=1$	Yen (1991)
O	$=\frac{\Sigma(n_iP_iR_i^{1/2})}{PR^{1/2}}$	Total shear velocity is weighted sum of subarea shear velocity	$\sqrt{gRS}=\Sigma\left(\frac{P_i}{P}\sqrt{gR_iS_i}\right)$ $(V_i/V)=(R_i/R)^{2/3}$	
P	$=\frac{\Sigma(n_iP_iR_i^{1/3})}{PR^{1/3}}$	Total shear velocity is weighted sum of subarea shear velocity	$\sqrt{gRS}=\Sigma\left(\frac{P_i}{P}\sqrt{gR_iS_i}\right)$ $(V_i/V)=(R_i/R)^{1/2}$	Yen (1991)
Z	$=\exp\left[\frac{\Sigma P_ih_i^{3/2}\ln n_i}{\Sigma P_ih_i^{3/2}}\right]$	Logarithmic velocity distribution over depth h for wide channel	$S=S_i, \quad Q=\Sigma Q_i$ $\frac{Q_i}{2.5\sqrt{gS}}=h_i^{3/2}P_i\left[\ln\left(\frac{10.93h_i}{k_i}\right)\right]$ $\frac{Q}{2.5\sqrt{gS}}=\Sigma h_i^{3/2}P_i\left[\ln\left(\frac{1093h_i}{k}\right)\right]$ $n=0.0342k$	Krishnamurthy and Christensen (1972)

By using the compound/composite roughness resistance factor, n_c, open-channel flow computations can be kept in the realm of 1D analysis, making a number of open-channel problems such as backwater curve calculations relatively simple without performing a more complicated 2D or 3D computation.

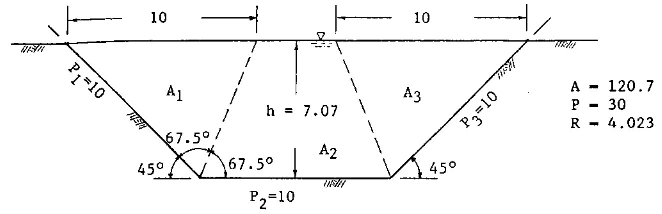
A number of formulas have been proposed for compound and composite channels based on different assumptions about the relationships of the discharges, velocities, forces, or shear stresses between the component subsections and total cross section. Seventeen of them are listed in Table 3. Furthermore, different methods have been suggested to divide the cross section into subsections for applications of these formulas, including:

1. Vertical lines extending from every break point of the geometry or boundary roughness up to the water surface;
2. Bisect lines of every angle at the geometric or roughness break points;
3. A horizontal or near horizontal line joining the two breaks of the channel at the bankfull stage, separating the compound channel into two parts: the lower main channel section and the upper flood channel section;
4. A variation of (3) above by further subdividing the lower main channel and upper channel by bisect angle lines or vertical lines;
5. Diagonal dividing straight lines or curves, with the intention to match the dividing lines as close to the zero-shear surfaces as possible (Yen and Overton 1973; Yen and Ho 1983);
6. A variation of (2) above when the bisect lines meet below the water surface is to extend the bisect meeting point vertically upward to the water surface from which the lines to the bisect points are used as the dividing lines.

Customarily, the internal water lines dividing the subsections are not considered as part of the wetted perimeter in computing the subsection hydraulic radius R_i. Only the actual wetted external boundary is used. This is equivalent to saying that the internal shear stresses at the dividing water lines are zero, which of course is not true except for very special cases. In the momentum or energy balance consideration for nonuniform flow in a compound or composite channel, convective transfer of the momentum or energy carried by the flow mass across the internal boundary dividing the subareas dominates the transfer due to interfacial shear on the subarea boundary (Yen et al. 1985) unless the flow is very close to a true steady uniform flow without secondary lateral flow. In other words even for steady uniform flow in a prismatic channel for which the water surface slopes are equal ($S_{wi}=S_w$), if secondary flow exists, the momentum or energy slopes are different from subarea to subarea, i.e., $S_{m1}\neq S_{m2}\neq\cdots\neq S_{mi}\neq S_m$ and $S_{e1}\neq S_{e2}\neq\cdots\neq S_{ei}\neq S_e$. In any subarea with lateral momentum and energy transfer, the momentum, energy, and water surface slopes are all different.

Which of the equations in Table 3 and which subsection division methods are more suitable for compound and/or composite channels, and whether better formulas can be developed from fluid mechanics remain to be investigated. A hypothetical comparison of the equations is made with the composite/compound channel depicted in Fig. 7, for which the floodplains extend laterally and horizontally 50 length units on either side of the main channel, with vertical walls at the far end of the floodplains. The Manning n for the floodplains and side walls are assumed 0.05. Variations of n_c with flow depth predicted by the equations in Table 3 for the example compound/composite channel using the vertical subdivision method are shown in Fig. 8.

The 17 equations can be grouped according to the geometry data required in applications. Equations I–P require both hydraulic radius and wetted perimeter. Hence, they depend on the way the subsections are divided in addition to the relative amount of the wetted perimeter of the subsections. Equations A–D require the relative area of the subsection. Hence, they depend on the method of subsection division but not at all on the wetted perimeter, not a physically reasonable situation, especially if n_c is viewed from the momentum perspective. Equations E–H require only the relative length of the wetted perimeter, independent of how the subsections are divided. Equation Z requires the wetted perimeter and the depth of the component channel flow, making it computationally cumbersome in application and suitable only for subdivision using verticals.



Subarea			Use bisect lines		Use verticals		
	n	P	A	R	A	R	$\bar{h}$
1	0.01	10	35.36	3.536	25.0	2.50	3.53
2	0.03	10	49.99	4.999	70.7	7.07	7.07
3	0.02	10	35.36	3.536	25.0	2.50	3.53

Fig. 7. Geometry of example composite/compound channel at bankfull level (from Yen 1991)

It has been found that the differences among the equations generally far exceed the differences due to subarea division methods. The very limited data available in the literature show considerable scattering on the plot, and they are insufficient to identify which equations are more promising. Besides, often it is unclear how the resistance values in the literature were computed in view of the possibilities shown in Fig. 6. Eventually, these equations should be assessed with accurate experimental data when they become available.

With the rapid development of computer technology allowing numerical analysis of flow in open channels in great detail, an increasing number of open-channel problems can now be solved by using 2D or 3D simulations instead of 1D solutions. However, for a significant amount of problems, the 1D approach remains the most efficient because of the data requirement and computational effectiveness, especially for real time operation for which lead time in forecasting is important. The 3D and 2D models require knowledge of point resistance coefficients or u_*/V in Fig. 6(a). The 1D models need the cross sectional or reach resistance coefficients. An alternative has been suggested by Yen et al. (1985) to apply the 1D flow equation to each of the vertically divided subareas and include terms to account for the lateral mass transfer across the vertical interfaces between subdivisions. In this approach, the basic n_i is applied directly to the subdivisions without computing the composite/compound n_c. Recently, Bousmar and Zech (1999) further demonstrate the feasibility of this method.

Vegetated Channel Resistance

Vegetation in channels covers a wide range of conditions, from highly flexible low grass blades to dense bushes to firm trees.

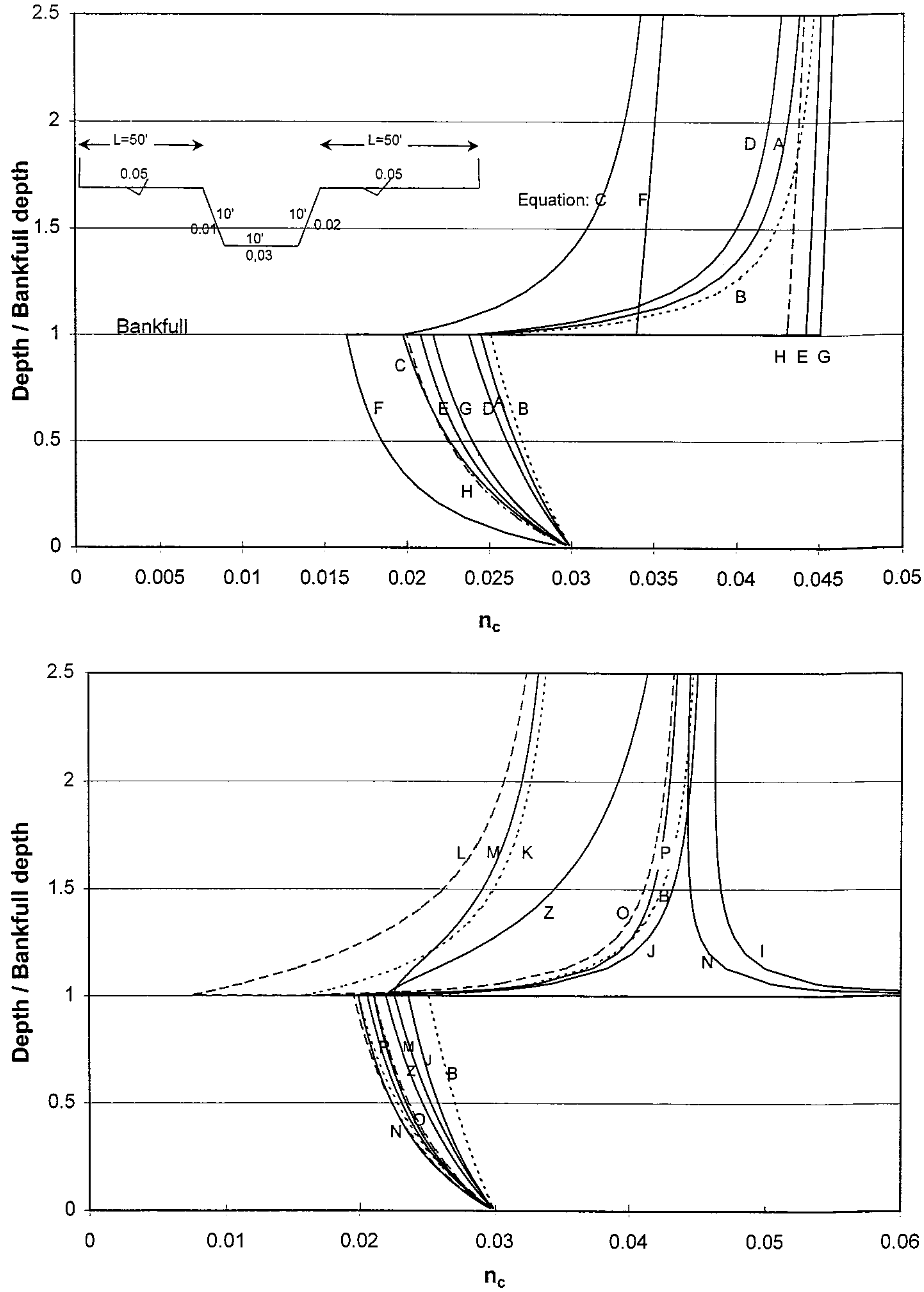


Fig. 8. Variation of Manning n_c with depth for example compound/composite channel

Presence of vegetation in the flow modifies the velocity distribution, and hence, the resistance. For vegetation extending from the channel bed, the symbolic roughness parameter **K** in Eq. (1) can be expanded to include symbolic nondimensional vegetation parameters $\mathbf{L}_v$ for representative geometry measure, **J** for its flexibility, **D** for its relative submergence, and **M** for its density distribution on channel bed. Thus, for steady flow over a wide (2D) channel, Eq. (1) can be rewritten as

$$f, \frac{n}{R^{1/6}} \text{ or } S = F\left(\mathsf{R}, \mathsf{F}, S_w, S_0, \frac{k}{R}, \mathbf{L}_v, \mathbf{J}, \mathbf{D}, \mathbf{M}\right) \qquad (32)$$

From the fluid mechanics viewpoint, an initial attempt to understand the physics of flow through vegetated channels is to study first the simpler case of steady uniform flow through rigid stubs distributed on the channel bed so that the parameters **J** and S_w need not be considered. Many early studies belong to this group, e.g., Li and Shen (1973). The next group of sources of information is from the many experiments on submerged grass (e.g., Chen 1976), flexible strips (e.g., Kouwen and Unny 1973) and trees.

The effect of interaction between the flexible vegetation and the flow is shown in Kouwen et al. (1981) and Kouwen (1992), among others. It has been generally agreed that vegetation increases flow resistance, changes backwater profiles, and modifies sediment transport and deposition. Information of typical previous studies on vegetation resistance can be found in Tables 4–6.

If the vegetation submergence is less than half of the flow depth, universal velocity distribution laws such as the logarithmic distribution may prevail in the upper nonvegetated part of the depth. Thus, the apparent shear velocity may be determined indirectly from the velocity distribution. For high submergence or protruding flexible vegetation, it is doubtful if the logarithmic

Table 4. Typical Investigations on Resistances of Vegetated Channels: Grassed or Artificial Strip Channels

Investigator(s)	Channel geometry	Grass type	Data	Approach	Dependent variable	Independent variables	Remarks
Ree and Palmer (1949)	Nearly trapezoidal	Different types of natural grass	Field	Curve fitting	n	VR	VR independent of channel slope or shape
Chen (1976)	Plane surface, seven different slopes	Bermuda or Kentucky Blue	Laboratory	Dimensional analysis and curve fitting	f	R, S_0	Also test flow under rain
Phelps (1970)	Plane	Articifical turf, mostly protruding	Laboratory	Dimensional consideration and data fitting	f	R, depth to side opening ratio	Flow in laminar or transition ranges. May not achieve uniform flow.
Kouwen et al. (1969)	Rectangular channel	Submerged flexible artifical strips on bed	Laboratory and field	Log velocity distribution and fitting	$\frac{V}{u_*}$	Cross sectional area A to vegetated area A_v ratio	
Kouwen and Unny (1973)	Rectangular channel	Submerged flexible artificial strips on bed	Laboratory	Dimensional analysis and log velocity distribution	$\frac{V}{u_*}$ or f	A/A_v, **J**, ratio of deflected height to flow depth	
Kouwen and Li (1980)	Plane or channel	Various	Four different sources	Log velociety distribution	n or f	**J**, depth, grass height, S	
Abdelsalam et al. (1992)	Four wide canals	Submerged weeds	Field	Empirical	n	Mean depth, or A/A_v	
Weltz et al. (1992)	Grass rangelands	Natural grass	Field	Kinematic wave model	f	Linear combination of various factors	

velocity distribution exists in either cycle-time averaged or instantaneous sense, and the resistance to the flow is probably more from the form drag of the vegetation than from the bed shear. In view of the highly variable interacting forces between the vegetation and the flow, as well as the increased energy losses due to the wakes behind the vegetation, one should be careful in selecting the energy or momentum concept as the effective means for analysis. Thus, it is understandable that much still remains to be done to establish the functional relationship of Eq. (32). Nevertheless, once the resistance coefficient for the vegetated part of a composite channel is determined, the channel cross sectional or reachwise resistance coefficient n_c can be computed by using one of the appropriate equations listed in Table 3.

Resistance of Sediment-Laden Channels

The major feature of alluvial and other sediment-laden channels concerning flow resistance is a boundary consisting of movable

Table 5. Typical Investigations on Resistance of Post-type Simulated Vegetated Channels

Investigator(s)	Channel geometry	Vegetation type	Data	Approach	Dependent variable	Independent variables	Remarks
Hsieh (1964)	Rectangular	Single row protruding circular cylinders, seven different spacings	Laboratory	Dimensional analysis, analytic and experimental	C_D	F and relative spacing	
Li and Shen (1973)	Rectangular	Distributed protruding circular cylinders	Laboratory	Analytic and experimental	C_D	Cylinder spacing and pattern	Also studied flow with sediment
Pasche and Rouve (1985)	Compound	Distributed protruding rods	Laboratory	Analytic and experimental	f	Rod density	
Dunn et al. (1996)	Rectangular	Distributed submerged rigid or flexible cylinders	Laboratory	Experimental and analytic	C_D	Several	
Lopez and Garcia (1997)	Rectangular	As Dunn et al.	Data from Dunn et al.	Analytic	n	Vegetation density	n increases with vegetation density

Table 6. Typical Investigations on Resistance of Nongrass Vegetated Channels

Investigator(s)	Channel geometry	Vegetation type	Data	Approach	Dependent variable	Independent variables	Remarks
Petryk and Bosmajian (1975)	Plane	Protruding trees or plants	Field	Analytic	n	Vegetation density	
Pasche and Rouve (1985)	Compound	Single row of protruding willow bush on flood plain	Field	Analytic	f	Vegetation density	
Kadlec (1990)	Plane	Protruding plants	Field	Theoretical	S	h, V	

particles, and hence, the formation of mobile bed forms and permission of water to flow through the voids between the particles. Accordingly, the velocity distribution and boundary shear near the bed are modified from those of similar rigid-boundary channels.

The channel bed forms can be loosely classified as plane bed, ripples, dunes, and antidunes. From the view point of resistance, the plane-bed channel is similar to plane rigid-wall channel in that the source of resistance is surface resistance. The major differences between the two are: (1) for a rigid impervious boundary, no water will penetrate the boundary, whereas for the sediment bed, water moves through the voids between the bed particles, and (2) for a sediment plane bed, some energy and momentum are spent on picking up, transporting and depositing the bed sediment. This is true even in the case of flow with equilibrium sediment transport for which the plane bed remains constant with respect to time.

For ripple-bed channels, the sources of resistance are from the form resistance as well as the surface resistance. For channels with dunes and antidunes, there is wave resistance in addition to the form and surface resistances. For antidunes, the water surface wave is in phase with the antidunes and the flow Froude number is high, around unity or greater, and hence, the relative contribution of wave resistance is more significant than for the case of dunes.

Bed form geometry is always three dimensional. The bed configurations migrate with sediment particles moving on their surface. For channels with finite width, the bed forms change across the channel, especially for compound channels and rivers with floodplains. Large bed forms behave similar to large wall roughness elements, and can be regarded as macroroughness. The nature of the roughness effects is best represented by the size, shape, and spatial distribution of the roughness elements (Rouse 1965). However, as the distribution of those elements becomes dense, the characteristic length of the roughness elements may be replaced by an equivalent value of the Nikuradse-type sand grain roughness, k_s. For large-scale fixed roughness elements, some relationships between the roughness element characteristics and the k_s value have been suggested. However, when bed form is present, a single value such as k_s alone, to represent all the effects of the size, shape, and spatial distribution of the roughness elements, is questionable.

When macroroughness elements are present, the expressions for the cross sectional and reach-average velocity distributions are more complicated. The local shear stress, velocity, and velocity gradient vary from point to point in a cross section and along a reach, even for a wide channel, due to the complex 3D flow patterns between roughness elements and possible flow separations. Accordingly, direct determination of friction slope using Eq. (21) is difficult since the integration over the cross section becomes complicated. This complexity makes computation of the ratio of the mean velocity to average shear velocity for a depth, V/u_*, very difficult.

With the presence of sediment on the channel bed and in the flow, the nondimensional symbolic relationship between the flow resistance and its influential dimensionless parameters from Eq. (1) is refined by expressing the roughness parameter **K** in terms of contributing nondimensional parameters of relative sediment size d_s/h, size gradation G, relative density $\Delta\rho_s/\rho$, sediment shape ξ, and suspended sediment concentration parameter C_s, where d_s = representative size measure of sediment, h = flow depth, ρ = density of fluid; $\Delta\rho_s = \rho_s - \rho$, where ρ_s = density of sediment. Thus, for a steady flow in a straight, prismatic, simple geometry, sediment-laden channel, the cross sectional flow resistance for either energy or momentum concept is

$$f, \frac{n}{d_s^{1/6}}, \text{ or } S = F\left(\mathsf{R}, \mathsf{F}, S_w, S_0, \eta, \mathbf{N}, \frac{d_s}{h}, \frac{\Delta\rho_s}{\rho}, \xi, G, C_s\right) \tag{33}$$

A discussion of the physical implication of the 11 independent contributing parameters in the right-hand side of Eq. (33) is given in Yen (1991).

Eq. (33) represents a complicated relationship between the 11 dimensionless independent parameters and the resistance coefficient. Rigorous studies on the relative importance of these parameters have yet to be performed. Such studies would require an extensive amount of new field and laboratory data for numerous flow and sediment conditions. Special measuring devices and consistent measuring techniques would be required for the collection of such data. The symbolic form of the independent parameters in Eq. (33) is only meant to be indicative. Many alternative forms of the nondimensional independent parameters have been proposed to replace those in Eq. (33). For example, the particle Froude number, particle Reynolds number, shear velocity to mean velocity ratio, and Shields' number have appeared in the literature. The selection and use of these alternative parameters depends partly on the specific conditions and partly on personal preference. These alternative forms of parameters can be obtained through cross multiplication and combination of the basic parameters given in Eq. (33), and these basic parameters can be replaced by the derived parameters on a one-by-one basis.

For a steady uniform flow in a straight, constant-slope, prismatic, rectangular channel with cohesiveless uniform-size spherical sediment such as the case of many laboratory flume tests, Eq. (33) can be simplied as

$$f, \frac{n}{d_s^{1/6}}, \text{ or } S = F\left(\mathsf{R}, \mathsf{F}, \frac{B}{h}, \frac{d_s}{h}, \frac{\Delta\rho_s}{\rho}, C_s\right) \tag{34}$$

where B = channel width. The bedform geometry parameters have not been included directly in Eqs. (33) or (34). For a steady uniform flow with equilibrium sediment transport, the bed form, similar to the resistance coefficient, is a dependent variable being a function of the nondimensional independent parameters in Eqs. (33) or (34). In such a case, the parameters C_s, S_w, and S_0 are no longer independent variables and they should be taken out from the functions F in this equation provided there is no hysteresis effect of bed form and channel morphology. The equilibrium sediment transport concentration C_s, like the bed form and resistance, is itself a dependent variable being a function of the same nondimensional independent parameters, whereas for steady uniform flow S_w, S_0, S_e, and S_m are all equal. For a stable bed form, the bed form geometry in a nondimensional form could be used to replace some of the nondimensional independent parameters in Eqs. (33) or (34), provided this geometry information is known.

For an unsteady flow with increasing discharge, the bed form may change with time as the flow changes. Eq. (33), adding the parameter **U**, is still applicable, with the bed form being a dependent variable replacing the resistance coefficients, while Eq. (34) is not applicable. Conversely, for unsteady flows with decreasing discharge, especially with graded sediment, the bed form is often determined by the high flow at an earlier stage of the flood. As the flood has receded, the relatively low flow is unable to move the large particles or the bed form settled earlier, and hence its geometry, which is incompatible with the current low flow condition, remains. After the flood, with a long time of low flow, some deposition of the smaller sediment may somewhat modify the previously formed bed geometry. But the major bed configuration from the early flow may persist. For such low flow cases, the bed form geometry should be entered as an independent variable directly in the nondimensional expression. This hysteresis effect of bed form offers a plausible explanation for the well known two-stage sediment rating curve.

If the channel is sufficiently wide and the sediment transport is in equilibrium—not an easy feat to accomplish in laboratory or in the field—Eq. (34) with constant $\Delta\rho_s/\rho$ can be simplified to three independent variables, as

$$S, \frac{n}{d_s^{1/6}}, \text{ or } f=F\left(\mathsf{R},\mathsf{F},\frac{d_s}{h}\right) \tag{35}$$

Any attempt to reduce this four-parameter problem into a three-parameter problem would require further justification of the particular situation to eliminate one of the three independent variables in the equation, R, F, and d_s/h (or their replacement alternatives). Dropping an independent variable without proper justification would make the analysis incomplete and confusing.

No theory has been developed to analytically define the function relating the channel resistance to its influential parameters, even for the simplest case of steady uniform flow in a straight, constant-slope, 2D wide channel with cohesiveless, constant-density, uniform-size spherical sediment under equilibrium transport as that indicated by Eq. (35). Presently, experimental data from the field and laboratories are the only means to provide a quantitative relationship of this four-dimensional (4D) function.

Linear Separation of Alluvial Channel Resistance

In this approach, the resistance coefficient is linearly separated into two parts

$$f=f'+f'' \tag{36}$$

or

$$n=n'+n'' \tag{37}$$

with the intention that each of the two parts is a function of only two of the three independent parameters in the right-hand side of Eq. (35) such that a 4D problem is transformed into two 3D problems. An obvious possibility is to assign the primed part for plane-bed resistance and the double-primed part for the bed-form addition. A list of selected representative investigations is given in Table 7. In Table 7, unless noted otherwise, steady uniform flow is assumed, cross sectional values are considered, and the momentum concept is followed. In data analysis, however, to establish equations or curves of the individual methods, reachwise values obtained from the energy relationship are actually used.

The linear superposition approach was initiated in Zurich, Switzerland (Meyer-Peter and Müller 1948; Einstein, 1950). They followed the momentum concept assuming the bed shear τ_0 can be separated linearly as

$$\tau_0=\tau_0'+\tau_0'' \tag{38}$$

in which τ_0' = reference plane-bed shear and τ_0'' = additional bed shear. Accordingly, the linearly separated component Weisbach resistance coefficients in Eq. (36) are

$$f'=8\left(\frac{\sqrt{\tau_0'/\rho}}{V}\right)^2 \tag{39}$$

$$f''=8\left(\frac{\sqrt{\tau_0''/\rho}}{V}\right)^2 \tag{40}$$

For Manning's n,

$$\frac{n'}{R^{1/6}}\frac{\sqrt{g}}{K_n}=\frac{\sqrt{\tau_0'/\rho}}{V} \tag{41}$$

$$\frac{n''}{R^{1/6}}\frac{\sqrt{g}}{K_n}=\frac{\sqrt{\tau_0''/\rho}}{V} \tag{42}$$

Hence, to satisfy Eq. (38), one obtains instead of Eq. (37), the following:

$$n^2=n'^2+n''^2 \tag{43}$$

There are two ways to define the reference plane-bed shear τ_0':

1. τ_0' is assumed as the same as the corresponding impervious rigid-plane boundary shear such as that given by the Moody diagram or Colebrook–White formula [Eqs. (17) or (19)], i.e., $\tau_0'=\rho f' V^2/8$ where f' is obtained from the Moody diagram using the given hydraulic radius and relative roughness.
2. τ_0' is assumed equal to that of a plane sediment bed of identical flow depth, velocity, and particle size but without any bed form.

In Case 1 τ_0'' consists of not merely the form drag but also the plane-bed shear difference between the sediment bed (Case 2) and the rigid bed (Case 1).

For a cross section, with τ_0 computed as γRS, linear separation of τ_0 can be accomplished through a linear division of the hydraulic radius or the slope S. For the former, (Einstein 1950)

$$R=R'+R'' \tag{44}$$

and hence,

$$\tau_0=\gamma R'S+\gamma R''S \tag{45}$$

For the latter (Meyer-Peter and Müller 1948),

Table 7. Selected Bed-Shear Based Linear Superposition Approaches to Alluvial Channel Resistance (from Yen 1991)

Investigator	Dependent variable	Independent variables	Data used[a]	Knowledge of bed required for application		Remarks
				Plane	Bedform	
		$R=R'+R''$				
Einstein and Barbarossa (1952)	V/u'_*	$\frac{d_{65}}{R'}, \frac{d_{65}u'_*}{11.6\nu}$	F	Yes		Assume log velocity distribution valid
	V/u''_*	$\frac{\Delta\rho_s}{\rho}\frac{d_{35}}{R'S}$			No	
Vanoni and Brooks (1957)	V/u'_*	$\frac{V^3}{g\nu S}, \frac{V}{\sqrt{gk_S S}}$		Yes		Modification of Einstein and Barbarossa on plane bed
	V/u''_*	Same as Einstein and Barbarossa			No	
Shen (1962)	V/u'_*	Same as Einstein and Barbarossa	L and F	Yes		Modification of Einstein and Barbarossa on V/u''_*
	V/u''_*	$\frac{\Delta\rho_s}{\rho}\frac{d_{35}}{R'S}, \frac{D_{50}V_T}{\nu}$			No	
		$S=S'+S''$				
Engelund and Hansen (1966, 1967)	V/u'_*	$\frac{\Delta\rho_s}{\rho}\frac{d_{65}}{R}$	L	No		
	V/u''_*	$\mathsf{F}^2\left(\frac{h}{L_{\text{dune}}}\right)\left(\frac{H_{\text{dune}}}{h}\right)^2$			No	
Simons and Richardson (1966)	$\frac{C'}{\sqrt{g}}-\frac{V}{\sqrt{gRS'}}$	$\frac{d_{85}}{h}$	L	Yes		
	$\frac{C''}{\sqrt{g}}-\frac{V}{\sqrt{gRS''}}$	$\frac{d_{85}}{h}, \frac{\Delta\tau_0}{\tau_0}$			Yes	
Vanoni and Hwang (1967)	f'	$\mathsf{R}, \frac{d_{50}}{R}$ (Moody diagram)	L	Yes		
	f''	R/(modified dune height)			Yes	
Alam and Kennedy (1969)	f'	$\mathsf{R}, \frac{R}{d_{50}}$	L and F	Yes		
Lovera and Kennedy (1969)	f''	$\frac{V}{\sqrt{gd_{50}}}, \frac{d_{50}}{R}$			No	
Acaroglu (1972)	f'	$\mathsf{R}, \frac{d_{50}}{R}$ (Moody diagram)	L and F	No		
	f''	$\frac{R}{d_{50}}$, $\phi=C_s\mathsf{F}\frac{R}{d_{50}}\frac{1}{\sqrt{\Delta\rho_s/\rho}}$			No	

[a]L—laboratory and F—field.

$$S=S'+S'' \tag{46}$$

thus,

$$\tau_0=\gamma RS'+\gamma RS'' \tag{47}$$

Note that linear superposition of f, Eq. (36), corresponds to

$$u_*^2=u_*'^2+u_*''^2 \tag{48}$$

The concept of Karman–Prandtl logarithmic velocity distribution and Nikuradse equivalent grain roughness apparently had considerable influence on the development of the estimation of the plane-bed roughness, bed shear, and resistance. For steady uniform flow in circular pipes with rigid impervious wall, Schlichting (1936) suggested $k_s=1.64\ d_m$, d_m being the mean diameter of the wall roughness elements. Colebrook and White (1937) gave a value $k_s=1.36\ d_m$ for mixed wall-roughness commercial pipes. Following the Nikuradse equivalent grain roughness concept, k_s is assumed proportional to a representating sediment size d_x

Table 8. Ratio of Nikuradse Equivalent Roughness Size and Sediment Size (from Yen 1991)

Investigator	Measure of sediment size, d_x	$\alpha_s = k_s/d_x$
Ackers and White (1973)	d_{35}	1.23
Strickler (1923)	d_{50}	3.3
Keulegan (1938)	d_{50}	1
Meyer-Peter and Muller (1948)	d_{50}	1
Thompson and Campbell (1979)	d_{50}	2.0
Hammond et al. (1984)	d_{50}	6.6
Einstein and Barbarossa (1952)	d_{65}	1
Irmay (1949)	d_{65}	1.5
Engelund and Hansen (1967)	d_{65}	2.0
Lane and Carlson (1953)	d_{75}	3.2
Gladki (1979)	d_{80}	2.5
Leopold et al. (1964)	d_{84}	3.9
Limerinos (1970)	d_{84}	2.8
Mahmood (1971)	d_{84}	5.1
Hey (1979), Bray (1979)	d_{84}	3.5
Ikeda (1983)	d_{84}	1.5
Colosimo et al. (1986)	d_{84}	3-6
Whiting and Dietrich (1990)	d_{84}	2.95
Simons and Richardson (1966)	d_{85}	1
Kamphuis (1974)	d_{90}	2.0
van Rijn (1982)	d_{90}	3.0

$$k_s = \alpha_s d_x \tag{49}$$

Some values of α_s obtained from the literature describing previous studies were reported in Yen (1991) and are reproduced in Table 8. The range of α_s values and the diverse representative sediment size used for d_x indicate further research on this concept is necessary.

Nonlinear Approaches to Alluvial Channel Resistance

In the nonlinear approach the resistance coefficient is not divided into grain roughness and bedform roughness as in the linear superposition approaches discussed previously. Instead, it is kept as a single factor. Most of the existing nonlinear approaches were derived from dimensional analysis and statistical fitting of data to the parameters considered in the functional relationships. Some representative investigations are listed in Table 9. They can be identified into four groups: (1) those considering the resistance coefficient as a dependent variable in their analysis; (2) methods based on shear stresses which can be related to resistance using Eq. (5); (3) those giving an equation of the mean velocity whose coefficients are related to resistance; and (4) the energy approach derived from Bagnold's (1966) stream power concept for expressions of velocity that could be used to calculate the resistance. Most of them implicitly assume the flow to be steady, reachwise uniform with equilibrium sediment transport. Unlike the linear separation methods, the use of many of the nonlinear approaches does not require knowledge of the bed configuration. Only Group (1) considers the resistance coefficients explicitly. The other three groups consider the resistance indirectly and implicitly.

Among the five direct nonlinear approaches to resistant coefficients shown in Table 9, Strickler's is for gravel beds without significant bed form. Only Camacho and Yen (1991) expressed the Weisbach resistance coefficient explicitly in equation form that is convenient for computer calculations. The remaining three showed their results in curves and graphs. Camacho and Yen's equations can be converted to Manning n form as

$$\frac{n}{R^{1/6}} = \frac{1.16}{c_n} \frac{T^{*0.175}}{\mathsf{R}^{0.19}} \quad \text{for } \mathsf{F} < 0.4 \tag{50}$$

$$\frac{n}{d_{50}^{1/6}} = \frac{0.054}{c_n} \left(\frac{d_{50}}{R}\right)^{-0.04} \frac{\mathsf{R}^{0.05}}{\mathsf{F}^{0.88}} \quad \text{for } 0.4 \leq \mathsf{F} < 0.7 \tag{51}$$

$$\frac{n}{d_{50}^{1/6}} = \frac{0.17}{c_n \mathsf{F}^{0.15}} \quad \text{for } 0.7 \leq \mathsf{F} < 1 \tag{52}$$

$$\frac{n}{d_{50}^{1/6}} = \frac{0.17}{c_n \mathsf{F}^{0.45}} \quad \text{for } 1 < \mathsf{F} < 2 \tag{53}$$

in which the dimensionless constant $c_n = 1$ for $K_n = \sqrt{g}$; $c_n = 3.132$ for SI units with $K_n = 1\ \text{m}^{1/2}/\text{s}$: and $c_n = 3.819$ for English units with $K_n = 1.486\ \text{ft}^{1/3}\text{-m}^{1/6}/\text{s}$ and d_{50} in ft; $T^* = S_w(R/d_{50})/(\Delta\rho_s/\rho)$, and S_w = water surface slope with respect to horizontal surface. Since the relationship between n and f is nonlinear, a direct regression of the data for n may yield a slightly improved result. Obviously, more in-depth analysis and more data covering broad ranges of sediment-laden channel conditions to obtain more reliable resistance coefficient formulas is most desirable.

Concluding Remarks

Flow resistance is an exciting subfield of hydraulics that is of practical importance and of intriguing fluid mechanics. Through the resistance coefficients, the complicated physical processes are represented succinctly for practical applications. Despite the success in the past, much can still be done in the future, such as investigating the effects of channel geometry and flow unsteadiness on the flow resistance.

The best way to understand flow resistance is through fluid mechanics, with the help of the universal velocity distribution laws of the theorems of boundary layer and turbulence. This approach offers a mathematical bridge between the wall skin friction and detailed physical transport process. Understanding the assumptions and limitations of the universal velocity distribution laws, including the logarithmic distribution, is desirable. Knowledge of energy dissipation and transfer by the fluid and the role of turbulence is helpful. Similar insight and understanding on the fluid mechanics involved in wave, form, and unsteadiness resistances are also important. The difference between the momentum concept and energy concept in viewing resistance should also be appreciated.

It appears that the significance of the space scale in resistance evaluation has not been sufficiently addressed. The differences between point, cross sectional, and reach resistance coefficients, such as those shown in Fig. 6, have yet to be investigated. The channels of natural alluvial rivers are usually nonprimatic and often curved. The local resistance varies from point to point and from cross section to cross section. While in 2D or 3D simulations of local phenomena, the use of local resistance is required, in 1D simulation of open-channel flows, it is the reachwise or cross sectional resistance coefficient that is practically useful.

Resistance to flow in sediment-laden channels remains a challenging topic for both research and application. Recently, the stochastic processes of sediment behavior near the channel bed are becoming better understood. The significance of the relative time scale of fluid velocity and bed-form movement velocity remains

Table 9. Selected Nonlinear Approaches to Alluvial Channel Resistance (from Yen 1991)

Investigator	Dependent variables	Independent variables	Data used[a]	Knowledge of bedform required for application
Based on Resistance				
Strickler (1923)	n	d_s	L and F	No
Yen and Liou (1969)	f	$\mathsf{F},\ \frac{d_{50}}{R},\ \mathsf{R}$	L	No
Mostafa and McDermid (1971)	$C_m=\frac{\sqrt{g}n}{K_n d_{50}^{1/6}}$	$\mathsf{F},\ \frac{d_{50}}{\delta}$	F and L	No
Griffiths (1981)	f	d_{50}/R or $V/\sqrt{g d_{50}}$	F	Yes (moving bed or not)
Camacho and Yen (1991)	f	$\mathsf{R},\ \mathsf{F},\ \frac{d_{50}}{R}$ or $T^*=S_w\frac{R/d_{50}}{\Delta\rho_s/\rho}$	F and L	No
Based on Shear				
Raudkivi (1967)	$\frac{V}{\sqrt{u_*^2-u_{*c}^2}}$.	$\frac{u_*^2}{\frac{\Delta\rho_s}{\rho} g\, d_{50}}$	None	Yes (sediment size)
Yalin (1977)	$\frac{C}{\sqrt{g}}$	$\frac{u_*^2}{\frac{\Delta\rho_s}{\rho} g\, d_s},\ \frac{u_*^2\, h}{g\, d_s^2}$	L	No
Brownlie (1983)	$\frac{RS}{d_{50}}$	$\frac{VR}{\sqrt{g\, d_{50}^3}},S,\sigma_g$	L and F	Yes
van Rijn (1984)	$\frac{C}{\sqrt{g}}$	$\frac{d_{90}}{R_b},\ \frac{h_{\text{bedform}}}{R_b},\ \frac{h_{\text{bedform}}}{L_{\text{bedform}}}$	L and F	Yes
Based on Velocity				
Garde and Ranga Raju (1966)	$\frac{V}{\sqrt{\frac{\Delta\rho_s}{\rho} g\, d_{50}}}$	$\frac{R}{d_{50}},\ \frac{S}{\Delta\rho_s/\rho}$	L and F	Yes
Based on Mixed Momentum and Energy				
White et al. (1980, 1987)	f	$\frac{\Delta\rho_s}{\rho}\frac{g}{\nu^2}d_{35}^3,\ \frac{u_*}{\sqrt{\frac{\Delta\rho_s}{\rho} g\, d_s}}$	L	No

[a] L—laboratory and F—field.

to be investigated. Little is known on the nonequilibrium sediment transport and its effects on the transport rate and resistance.

The concurrent existence of many different linear and nonlinear approaches for the determination of alluvial channel resistance is, in fact, a reflection of the complexity of this problem and an indication that much has yet to be done to achieve a satisfactory solution. Alluvial channel resistance comes from a combination of surface, form, wave, and unsteady resistance components. The resistance varies from point to point, especially for composite and compound channels, whereas the resistance coefficient for a cross section or a reach of a channel is a weighted spatial average of the local resistance. As indicated in Eq. (33), there are many parameters contributing to the resistance coefficient. Presently, our concept of resistance is primarily an extension of existing knowledge on impervious, rigid, circular pipes or wide channels, which are different from the movable boundary of alluvial channels. Perhaps a more satisfactory approach to determine the resistance coefficient can be accomplished when improved understanding is achieved on the boundary layer over movable beds, on the chaotic behavior of turbulence and its influence on the velocity distribution and sediment transport, and on the effects of the bed forms and suspended sediment concentration on the flow.

In the linear superposition approach, caution must be exercised if there are more than one supplementary resistance factors to be added to the basic reference f' or n', i.e., $f=f'+\Sigma f_j''$ or $n=n'+\Sigma n_i''$ when $i>1$, such as the case of n combination suggested by Cowan (1956) and often quoted in the literature. Values of n_i'' are generally determined by separating each n_i'' individually from n' without considering the effects of other $n_i''s$. Since the relationship among the various factors is nonlinear, a linear combination using n_i'' values so determined usually overestimates the combined value of n.

As to the preference of using Manning n or Weisbach f, Eq. (5) shows that they are interchangeable and theoretically equivalent. However, in a practical sense for fully developed turbulent flow, Manning n has often been regarded as near constant for a given type of rough boundary surface. If further investigations confirm this attribute, the local surface roughness n can be used as the basic values to form the cross sectional n values of the composite or compound channels, by using appropriate equations such as those in Table 3.

Much on channel resistance has been learned since Rouse published his insightful paper in 1965. But it is still far from meeting his challenge of establishing a working formula in the form of Eq. (1). Hopefully, this presentation will stimulate new interests in this direction.

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