Two-Dimensional Modeling of Floods to Map Risk-Prone Areas

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Abstract: The assessment of flood risk is a difficult task and usually requires solution of a flood routing problem as a part of the assessment. Due to the large number of scenarios that have to be developed and analyzed, simplified numerical models are used for the computation of flooded areas in each scenario. More sophisticated models are often too complex to manage or, due to their design generality, not well suited to deal with the specific needs of flood routing problems. A comparison among three different models, with varying degrees of simplification and abstraction, is presented and discussed. Some considerations on the effectiveness of simpler models are drawn, focusing on the prediction of flooded areas.

DOI: 10.1061/(ASCE)0733-9496(2002)128:3(168)

CE Database keywords: Two-dimensional models; Flood routing; Flood plains; Italy.

Introduction

Every river control project should be examined during the design phase to determine its potential effects with regard to flooding. This requirement is imposed by regulations in Italy and in many other countries. It is important because flood risk assessment is a preliminary step to land use planning and to flood contingency planning and management. Local agencies usually have to produce and maintain maps of flood-prone areas where stricter regulations on land use may apply. In determining flood risk, the hydrological aspects are often the most important factors, but hydraulic routing processes also play a significant role. Only the latter is discussed here, in amounting to solution of a time-dependent, free-surface flow field with irregular boundaries over a large area.

Flooding may occur due to a variety of reasons, among which notably are dam break, levee failure, and levee or bank overtopping. To fulfill the requirements set forth for approval of river control works, a large number of diverse flooding scenarios need to be considered with the flooded areas for each determined through suitable calculations. This is seldom computationally feasible with 3D models, mostly due to difficulties involved in simulating flooding processes with adequate precision, as compounded by the large number of cases that should be considered. These difficulties are partly related to the collection of detailed data required by the more accurate models, but also because these models are often large scale and difficult to master. The resources required to perform such analyses are normally unavailable in engineering practice. As a result, simplified models are often employed, making it important to understand the inaccuracies that can arise due to model simplification.

Presented herein is a comparison of three models, as described in Greco et al. (1999), called FIVFLOOD, PA-31, and MCEP, an Italian acronym. The comparison is not among the specific models, because each represents an entire class of simplified 2D or quasi-2D models. FIVFLOOD is a complete model such as TABS-MD WES RMA2 (2001), just to name one that is better known worldwide; PA-31 is a parabolic model, similar to FLO-2D (O'Brien et al. 1993), and MCEP is a cell model similar to the ROUTWEIR model of Laura and Wang (1984). These models are compared for accuracy, efficiency, and conceptual simplicity. The accuracy comparison focuses only on the flooded area prediction, considering the areal extent of the flooded area and its shape. Three test cases are selected for assessing the performance of the models, with two of them being floodplain inundation from a nearby unleveed channel, and the third focusing on flooding that may occur as a consequence of levee failure.

Recent Contributions to Numerical Flood Routing

Published comparisons among different models can be classified into three categories:

1. Comparison of various numerical schemes for solving the same form of equations;
2. Comparison of 1D versus 2D models; and
3. Comparison of models derived with different degrees of abstraction and simplification of the complete equations.

Category 1 may be considered as a subset of 3, but is treated autonomously here due to the large number of papers that comprise it. With this prespective, only a few papers involve category 3 comparisons, mostly for 1D approaches. Because the authors believe these comparisons could be better addressed from a practical standpoint, a further contribution is provided herein to the understanding of what is gained or lost by use of simplified models in 2D flooding simulations.

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Note. Discussion open until October 1, 2002. Separate discussions must be submitted for individual papers. To extend the closing date by one month, a written request must be filed with the ASCE Managing Editor. The manuscript for this paper was submitted for review and possible publication on June 21, 2000; approved on July 17, 2001. This paper is part of the ***Journal of Water Resources Planning and Management***, Vol. 128, No. 3, May 1, 2002. ©ASCE, ISSN 0733-9496/2002/3-168–178/$8.00+$.50 per page.

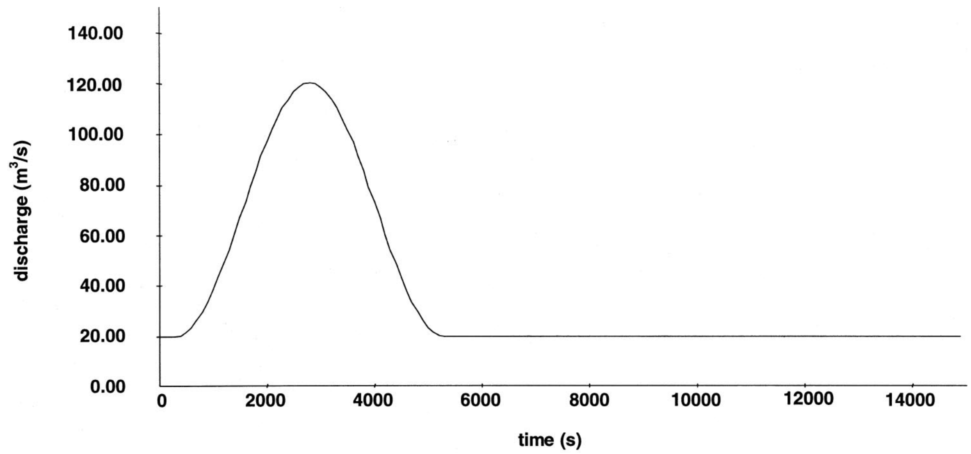

Fig. 1. Inlet hydrograph

Referring to the TELEMAC-2D finite element software, two alternative numerical schemes for the advection step are discussed by Bates et al. (1994) as an example of category 1 for evaluating the effects on discharge prediction. For category 2, O'Brien et al. (1993) compared the results of the 2D model FLO-2D (based on a diffusive wave approximation) with those of the 1D model HEC-2 for steady flow in a variable area channel. The authors found a good agreement between the two models.

Bladè et al. (1994) developed a quasi-2D numerical model for flood routing in rivers and floodplains. The main river is modeled as unsteady 1D flow, while the floodplain is represented by several interconnecting storage cells with water level calculated using mass conservation equations. This model, as validated against some theoretical examples, was compared with other commonly used, albeit crude, approaches: steady gradually varied flow and integration of the 1D de Saint Venant equations. In these other approaches, the flow cross sections include both the river and the floodplain, and a single water level value is used for the entire cross section. The authors found that the predicted time of occurrence of maximum depths varied more than the actual maximum depth.

A 2D model for providing more accurate prediction of flood wave propagation in the main channel and floodplain simultaneously, as well as a 1D model for simulating flood flows along the river, was developed by Tingsanchali and Dey (2000). In conducting sensitivity analyses, the authors observed that for the 1D model, Manning's has a significant effect on computed discharges, but little effect on computed water levels. For the 2D model, the weighting factor of the effective stresses, the friction factor of the main channel and berm, and the time step have a significant effect on the computed results. After applying both models to compute a flood in the river Ganges, Bangladesh, they found that the 2D model provided better accuracy when compared with actual measured flows.

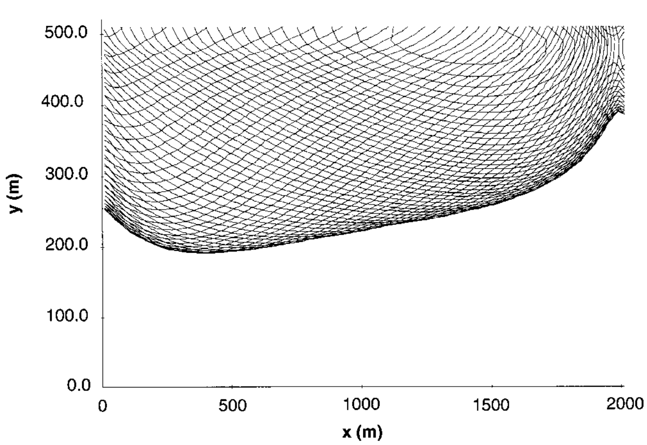

Fig. 2. Flooded area envelope computed by FIVFLOOD

Juza and Barad (2000) compared three numerical models simulating riverine systems for a case study and also discussed the effects of discretization of the flow field. The study includes a steady-state 1D model (WSPRO), an unsteady looped model (MIKE 11), and an unsteady 2D model (MIKE 21). The results of the comparison led the authors to observe that, given the availability of sophisticated tools and easy access to high performance computing platforms, the more complex and precise models should be used. However, in their opinion, not every problem requires use of the more advanced capabilities. Conversely, it is crucial for users to be aware that all models have assumptions and limitations and to make sure that the chosen model does not significantly misrepresent the system in regard to the intended objectives.

According to McCowan and Collins (2001), when a higher degree of accuracy in flood modeling than is possible with conventional 1D and "quasi-2D" modeling, is required, as in urban flood prediction systems, it is necessary to use full 2D modeling. They describe the advantages of 2D modeling using MIKE 21 relative to the more conventional approach of using branched 1D models, as well as the improvements in technology that have made this approach feasible. For the case study, differences in peak flood levels and flows between the 1D and 2D models were found to be quite significant.

As for the few references in category 3, the classical paper by Cunge (1969) showed how the Muskingum method, developed as a conceptual model, could be viewed as a special case of a parabolic approximation to the de Saint Venant equations. Another example is the work by Cappelaere (1997), who compared the results of a new formulation of the diffusive wave simplification of the unsteady, open-channel flow equations against those obtained by the full de Saint Venant equations. The test cases were a regular channel and an actual river reach with simulated discharges used for the comparison. The authors concluded that the simplified model can be safely applied to a wide range of flow conditions, while complying with the practical constraints of

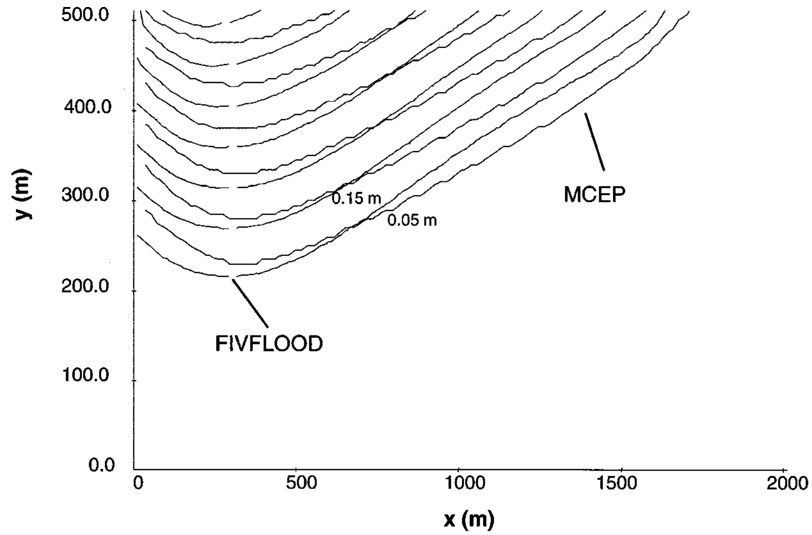

Fig. 3. Iso-depth contours at $t = 3{,}000$ s (FIVFLOOD versus MCEP)

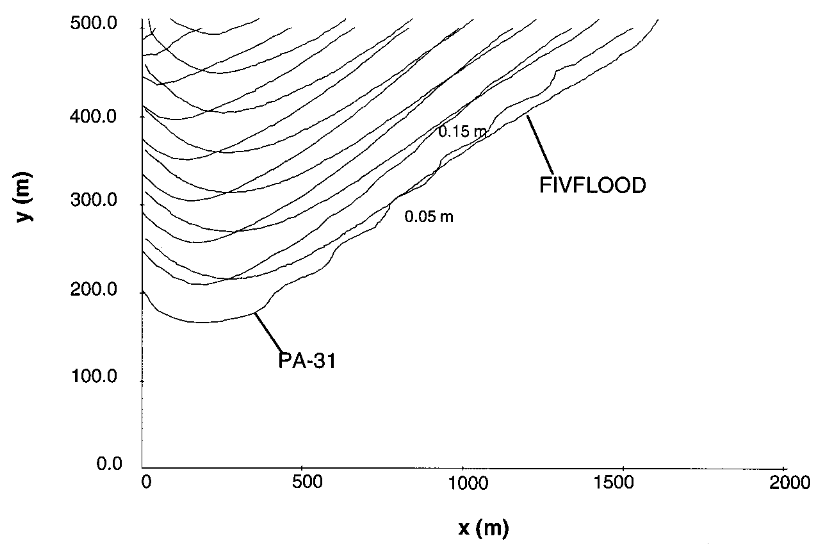

Fig. 4. Iso-depth contours at $t = 3{,}000$ s (FIVFLOOD versus PA-31)

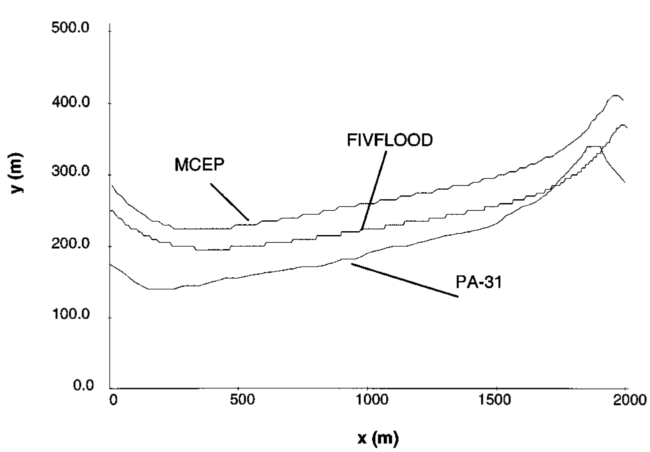

Fig. 5. Flooded areas envelopes

flood routing applications, including the usual unavailability of adequate channel geometry data. Molinaro et al. (1994) tested their code, a simplified 2D model based on a parabolic approximation, and discussed its capabilities to simulate real world flooding events after an evaluation of models solving the complete form of the shallow water equations.

Presented herein is a comparison of type 3 among models all solving a 2D form of the shallow water equations. The particular focus is on the prediction of the flooded surface shape and area, which appears not to have been sufficiently addressed in a detailed manner in previous studies.

Selected Numerical Models

If flooding is essentially regarded as a 2D process, the shallow water equations in conservation law form can be assumed to hold (Lai 1977)

$$\frac{\partial h}{\partial t} + \frac{\partial uh}{\partial x} + \frac{\partial vh}{\partial y} = 0 \tag{1}$$

$$\frac{\partial}{\partial t}(uh)+\frac{\partial}{\partial x}\left(u^2h+g\frac{h^2}{2}\right)+\frac{\partial}{\partial y}(uvh)=gh(S_{0x}-S_{fx}) \quad (2)$$

$$\frac{\partial}{\partial t}(vh)+\frac{\partial}{\partial x}(uvh)+\frac{\partial}{\partial y}\left(v^2h+g\frac{h^2}{2}\right)=gh(S_{0y}-S_{fy}) \quad (3)$$

where x,y = space coordinates; t = time; g = gravity acceleration; h = water depth; u = velocity component in the x direction; v = velocity component in the y direction; S_{0x} = bottom slope in the x direction; S_{0y} = bottom slope in the y direction; S_{fx} = friction term acting in the x direction; and S_{fy} = friction term acting in the y direction.

In these equations, internal friction effects are neglected, and only bottom stresses are accounted for. It is assumed herein that accurate predictions of flooding can be obtained by solving Eqs. (1)–(3) with appropriate boundary conditions.

Because no analytical solution of Eqs. (1)–(3) exists, approxi-

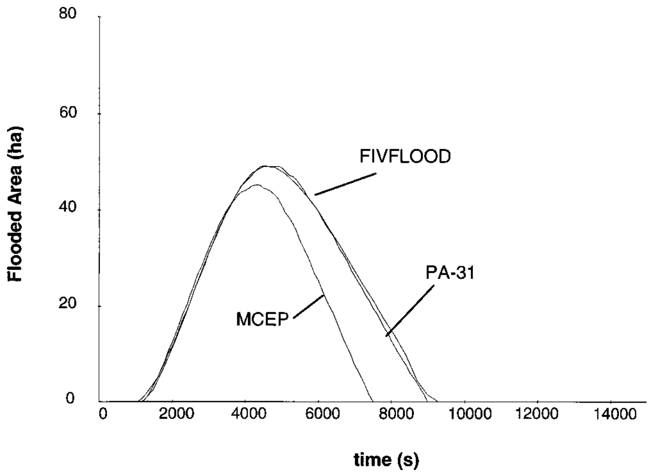

Fig. 6. Flooded area versus time

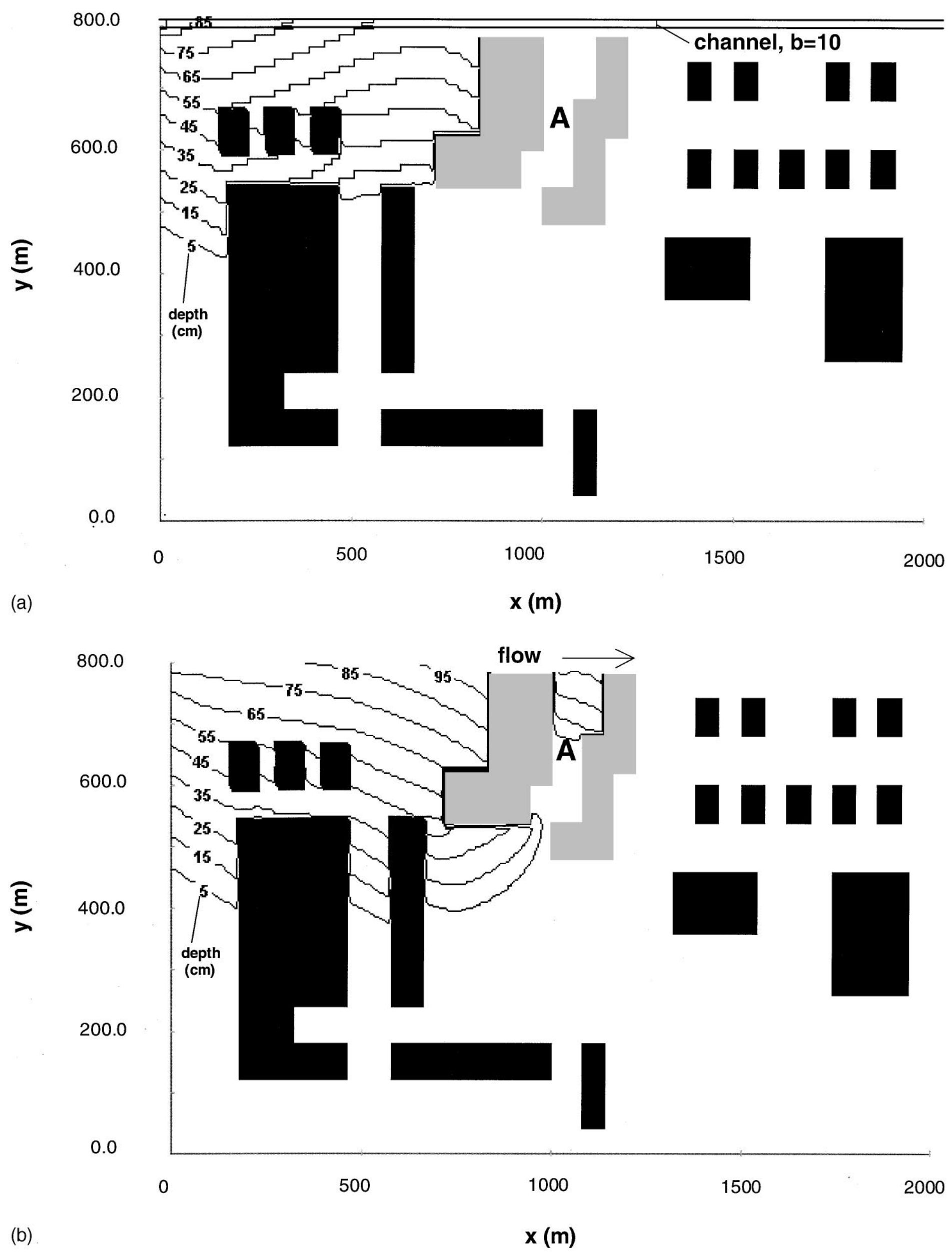

Fig. 7. (a) Iso-depth contours—PA–31 (urban area test)—$t = 3{,}000$ s; (b) Iso-depth contours—PA–31 (urban area test)—$t = 4{,}000$ s

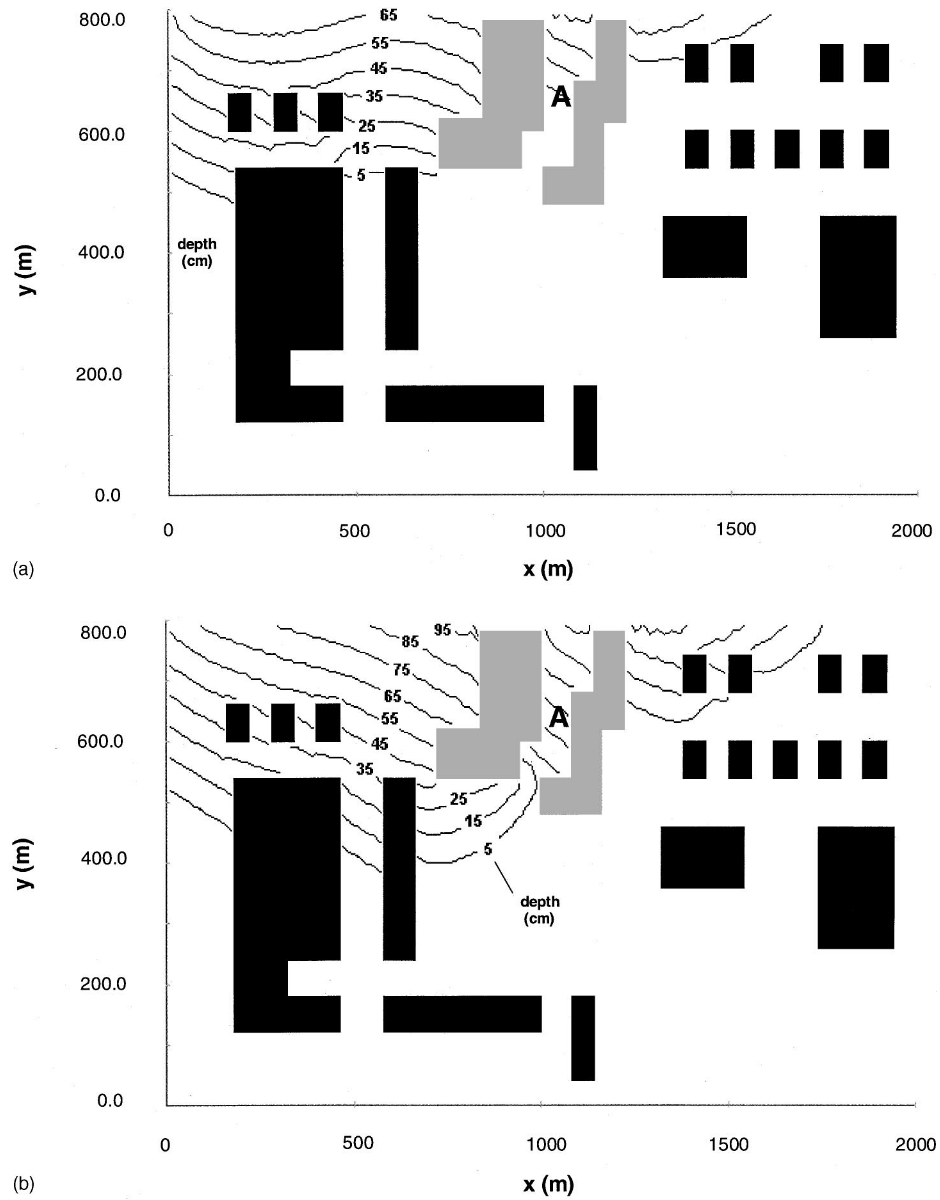

Fig. 8. (a) Iso-depth contours—FIVFLOOD (urban area test)—$t = 3{,}000$ s; (b) Iso-depth contours—FIVFLOOD (urban area test)—$t = 4{,}000$ s

mate solutions must be obtained by numerical methods. The accuracy of the solution depends primarily on the quality and quantity of data available to describe the study area. Sometimes problems do exist about the determination of the boundary conditions needed for the problem, and accurate boundary conditions are generally difficult to define. Another source of difficulty in the numerical solution is the irregular shape of most natural watersheds.

FIVFLOOD

The FIVFLOOD code has been used in this paper to solve the complete form of Eqs. (1)–(3)—the shallow water equations. This code is based on the finite volume formulation of the equations. A distinguishing feature of FIVFLOOD is that it couples a 1D model for the main channel with a 2D model for the valley floodplain. This coupling transfers mass and momentum between the two models without introducing simplifications, and frees the user from the need to specify the main channel as a part of the 2D simulation, which is often a source of numerical and practical difficulties. The authors' experiences with other industrial or commercial codes has shown a large use of computational resources for simulation of flooding by bank overtopping due to the constraints of having the main channel as a part of the general 2D simulated area. Calculations by other authors of flooding phenomena have required similarly large computation times.

PA-31

Two additional simplified models have been compared with FIVFLOOD. The first, PA-31, was developed at the Univ. of Palermo by Oliveri et al. (1996) with the aim of simulating flood propagation in urban areas. Its foundation relies on the concept of approximating the flooded area with a network of rectangular channels. The nodes at extreme sections link the channels together. In each node, a ''submersible'' area is considered where storage effects are assumed to be concentrated. While developed for urban areas, where blocks and streets actually constitute a network of

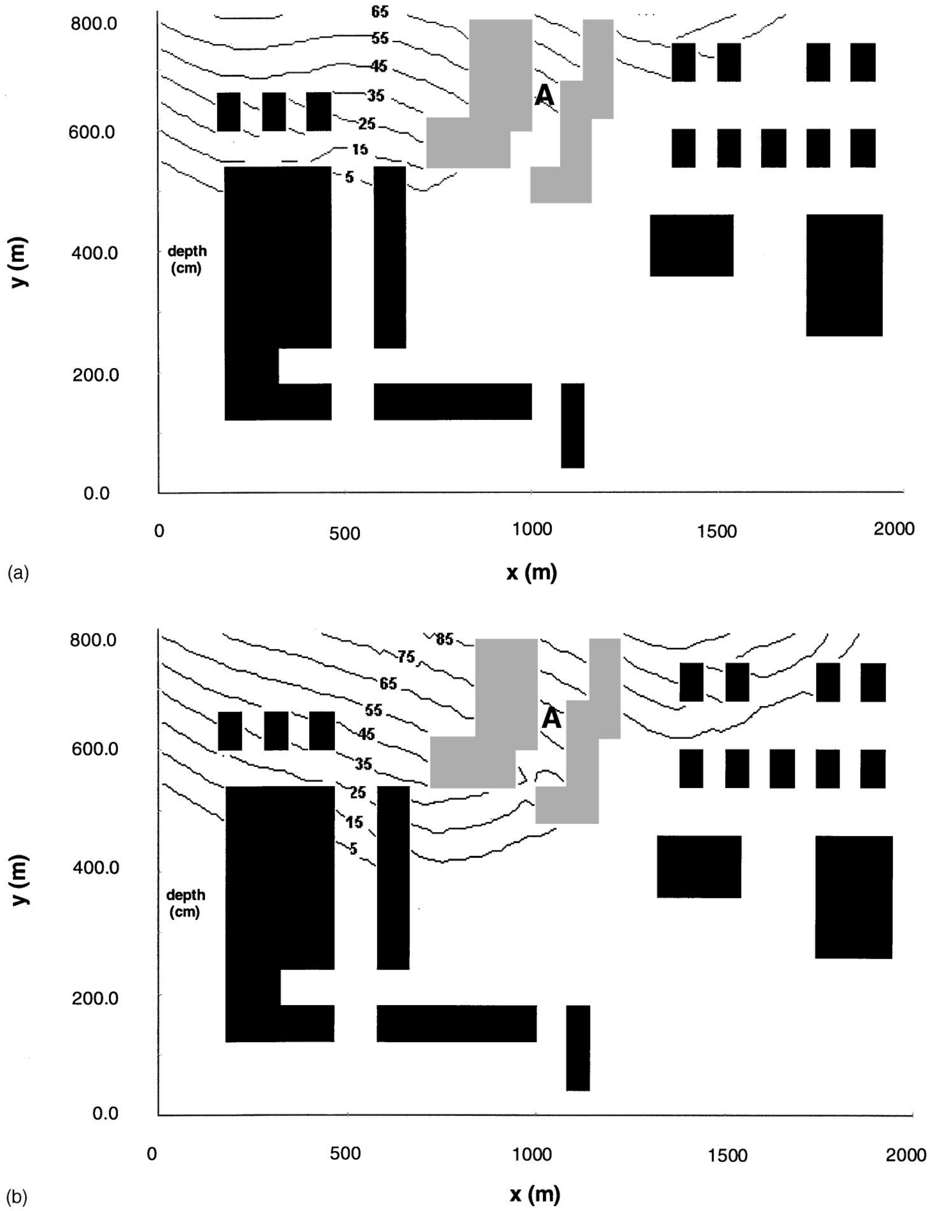

Fig. 9. (a) Iso-depth contours—MCEP (urban area test)—$t = 3{,}000$ s; (b) Iso-depth contours—MCEP (urban area test)—$t = 4{,}000$ s

channels, it can also be assumed as a semiconceptual model for flooding in open areas. It was successfully applied to simulation of flooding that occurred in Palermo in 1931; the computed flood depths had a good agreement with the recorded ones. The same concept has been used previously by Natale et al. (1990), and similar models have been used to simulate the inundation in 1966 of Florence and to identify flood prone areas inside the city of Sondrio.

In PA-31, the 1D de Saint Venant equations are solved by a parabolic approximation (Cappelaere 1997; Ponce et al. 1978; Strelkoff and Katopodes 1977) for each channel, while only continuity equations are solved at the nodes. An implicit numerical scheme solves the nonlinear equation system for every time step; for unknowns, the water elevation at the channel network nodes, once water levels at nodes are known an explicit formula gives the discharge in each channel. Solution of the nonlinear system is carried out by an iterative Newton-Raphson method, obtaining an algebraic system with a sparse, diagonally dominant, symmetric, and positive definite Jacobian matrix. The corresponding linear system can be solved efficiently using a reduced amount of computer storage.

MCEP

The second simplified model, called MCEP, is truly bidimensional and is derived under a finite volume approach. The entire computational domain is divided into quadrilateral cells with the interface between two cells being real (e.g., a levee or a cliff) or ideal. The model uses the continuity Eq. (1), but assumes that the momentum Eqs. (2) and (3) can be replaced by a weir type formula for the computation of flows between cells (Greco et al. 1999).

This approach has been considered before, and among the proposed applications one can find the Mekong River delta model (Zanobetti and Lorgere 1968) and the ROUTWEIR model (Laura and Wang 1984) used to simulate the Rio Culebrinas (Puerto

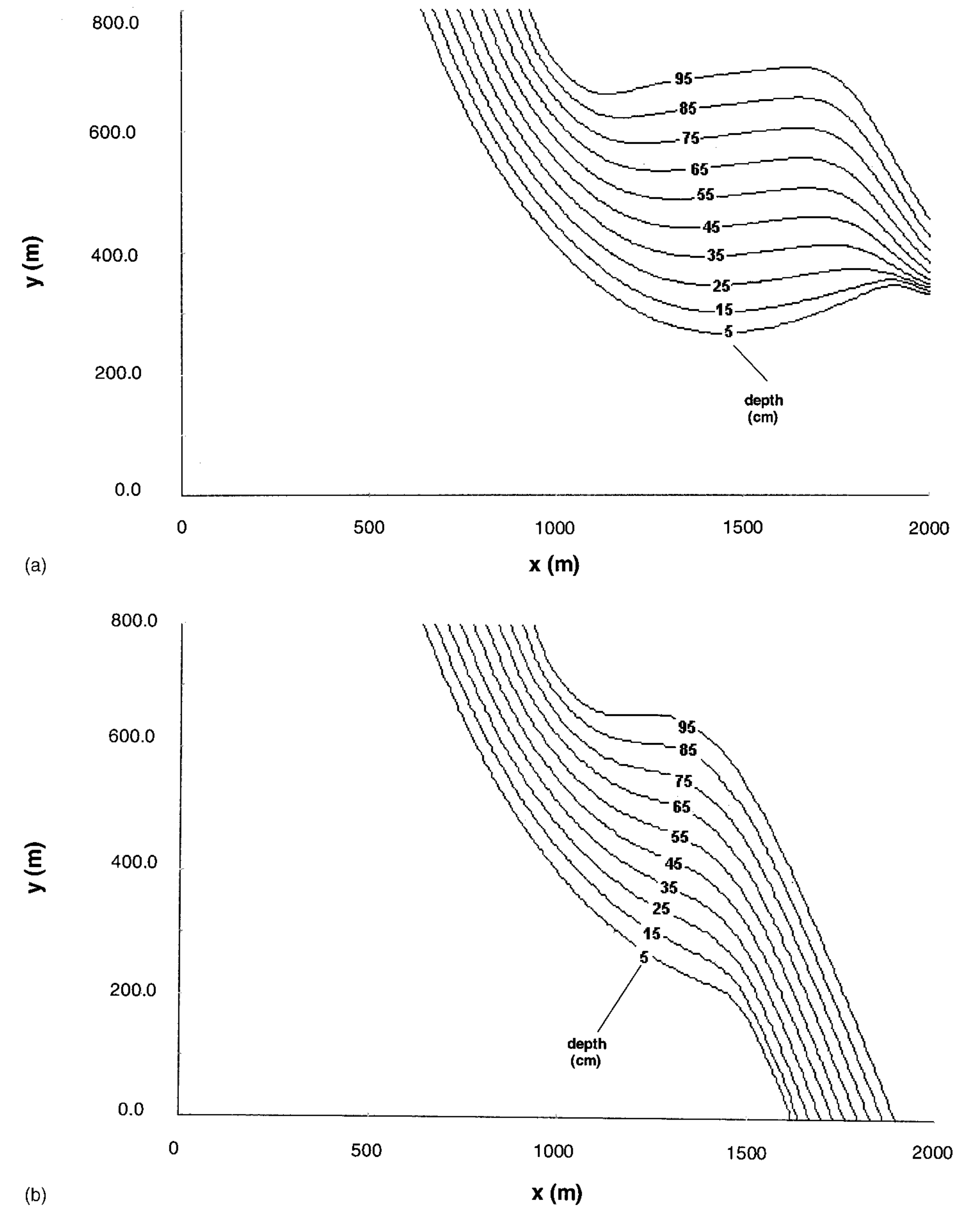

Fig. 10. (a) Iso-depth contours for the levee failure test—PA–31—$t=1{,}000$ s; (b) Iso-depth contours for the levee failure test—PA–31—$t=2{,}000$ s

Rico) flooding. An application of the latter was also proposed by Maione et al. (1986) to study the effect of a highway on the flood phenomena. Another similar model (Bladè et al. 1994) was proposed for flooding of large areas.

As in FIVFLOOD, the main channel is modeled with the full dynamic 1D de Saint Venant equations. An interface, again similar to that used in FIVFLOOD, connects the main channel with the valley portion of the model. A weir flow type of formula is applied at cell boundaries to compute mass fluxes between cells. This could then be seen as a crude parabolic approximation for Eqs. (2) and (3). In a recent paper (Han et al. 1998), a 2D parabolic model was presented in which the final formulas for computing mass fluxes between cells have the same structure.

The main differences between these two simplified models lies in treatment of the momentum equations. The PA-31 model assumes they can be simplified, while still preserving their original meaning, and implicitly assumes that bottom slopes can be considered constant along each channel reach. The solution is obtained by an implicit discretization, which allows use of large time steps without the need for satisfying a Courant-like condition but requires the solution of a nonlinear system of equations. The more conceptual MCEP model replaces momentum equations by explicit formulas valid for weir type flow. This approach also assumes a stepped bottom, which is not unusual for terraced valleys but is considered an approximation in other cases.

The dynamic effects of 2D flow should be better represented by the Oliveri PA-31 model at the added cost of a slightly greater complexity of the model. For real world problems, any model should be calibrated against experimental data. Model calibration usually involves adjusting the bottom friction coefficients, starting from the values suggested in engineering handbooks. Here, it is assumed that the Strickler coefficient has already been determined. Due to its conceptual nature, no guidelines exist for the initial assignment of the MCEP efflux coefficient.

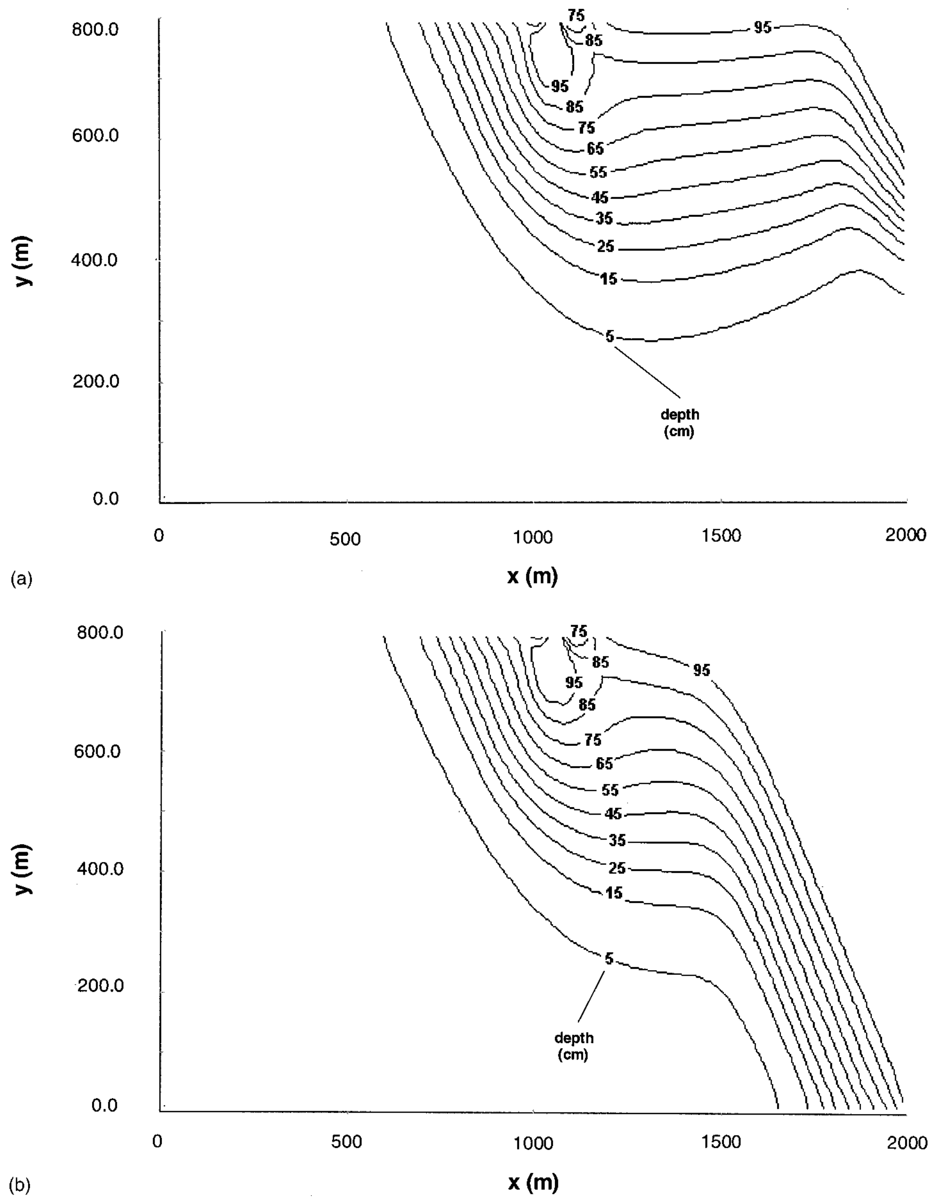

Fig. 11. (a) Iso-depth contours for the levee failure test—FIVFLOOD—$t = 1{,}000$ s; (b) Iso-depth contours for the levee failure test—FIVFLOOD—$t = 2{,}000$ s

Test Cases

Two out of the three test cases involve the flooding of a schematic rectangular floodplain valley of sides $Lx = 2{,}000$ m, $Ly = 800$ m with constant slopes $Sbx = 0.0008$ and $Sby = 0.002$ in the x and y directions, respectively. In cases 1 and 2, an unleveed rectangular channel 10-m wide flanks the valley in the x direction. The channel bottom is 3 m below the bank. A fixed depth is assumed at the downstream section of the channel, $hv = 2.69$ m. The inflow discharge in the channel rises, as depicted in Fig. 1, from 20 m^3/s to 120 m^3/s and then returns to the initial value. All other boundaries are assumed to be impervious. Case 2 differs from case 1 by the presence of secluded areas in the valley that cannot be flooded. These areas may represent buildings, wall surrounding blocks, or localized high topography.

Case 3 refers to a valley of identical shape and surface, with $Sbx = 0.003$ and $Sby = 0.002$, in which a levee protects the floodplain along the x direction. Flooding caused by the sudden collapse of a section of the levee is simulated by assuming a fixed depth of 2 m at $y = 800$ m and x between 1,030 m and 1,060 m with the remaining boundaries being impervious.

Performance of the Simplified Models

The solution given by FIVFLOOD is assumed to be the most accurate, and is taken as the reference solution against which are compared the solutions obtained by the simpler models. A spacing $\Delta x = \Delta y = 20$ m, with a time step of $\Delta t = 0.1$ s has been used. The effects of different selections of computational step sizes in FIVFLOOD have been investigated and found to be negligible. Both simplified models have been applied to test case 1, which is the simpler among the tests considered.

The PA-31 model is tested assuming the same Strickler roughness coefficient used for computations with the FIVFLOOD model. No separate calibration of the model PA-31 was thus per-

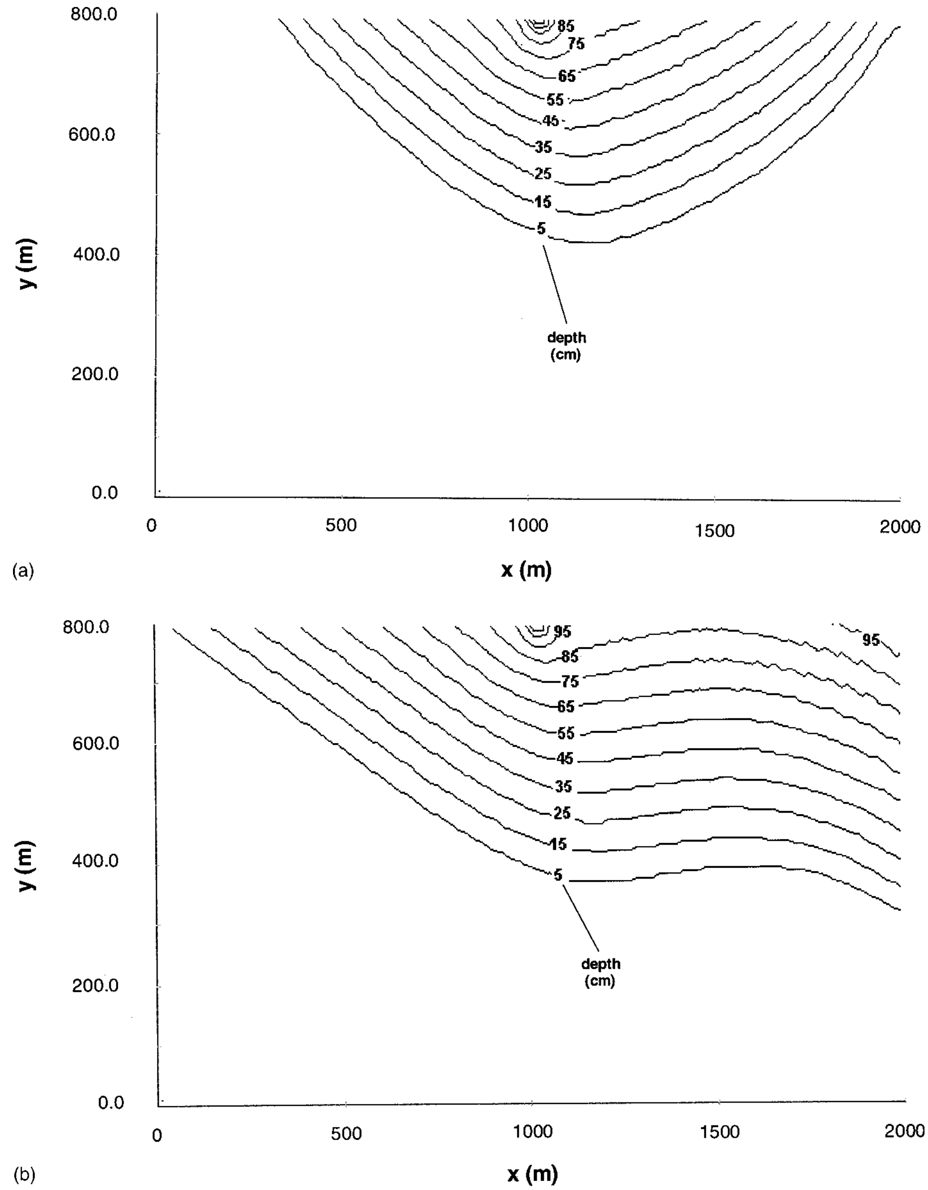

Fig. 12. (a) Iso-depth contours for the levee failure test—MCEP—$t=1{,}000$ s; (b) Iso-depth contours for the levee failure test—MCEP—$t=2{,}000$ s

formed. The node spacing for this model was assumed variable and, on average, somewhat larger than that used for the two other models.

MCEP uses the same finite volumes grid used by FIVFLOOD. It has been calibrated heuristically, assuming a single efflux coefficient to be applied to all cells, namely $m=0.90$. Sensitivity of MCEP results to the calibration parameter m has been investigated, and appears to be rather low.

The simplified models are compared against the reference solutions with focus primarily on the flooded area prediction rather than on the comparison of predicted water velocities and depths. An area is considered to be inundated if the average water depth reaches at least 0.05 m to exclude areas with spuriously low water depths.

In the first test case, the cumulative flooded area envelope during the event computed with FIVFLOOD was 574,000 m^2; its shape and inundation limit are shown in Fig. 2. With MCEP, a flooded surface area of 510,400 m^2 was obtained, while PA-31 gave an estimate of 609,000 m^2. Fig. 3 compares the water depth contours predicted by FIVFLOOD versus MCEP at $t=3{,}000$ s. Fig. 4 compares the depth contours at $t=3{,}000$ s as computed by FIVFLOOD and PA-31. The predicted contours of the cumulative flooded area for each model is shown in Fig. 5. Fig. 6 shows a comparison between the three models of the flooded surface areas versus time.

As can be seen by the figures, both the simplified models are capable of representing the main characteristics of the flooding process. While the total area of flooding predicted by PA-31 follows more closely that of FIVFLOOD over time (Fig. 6), differences still exist in the area shape, and the cumulative flooded surface is predicted as larger than actual (Figs. 4 and 5). Instead, MCEP underestimates both the instantaneous flooded areas and the cumulated one (Figs. 3, 5, and 6) so that its use for predicting flood risk may be less conservative, at least in this test case where dynamical effects seem to have a significant role. Maximum predicted depths are, however, better computed by MCEP than by PA-31 and tend to err on the conservative side.

The second test case, as stated previously, differs from the first only by the presence of obstacles on the floodplain. It can represent an urban area with impervious buildings. Figs. 7, 8, and 9 report the comparison among PA-31, FIVFLOOD, and MCEP (results for this case at time $t=3{,}000$ s and $t=4{,}000$ s). As can be seen, PA-31 underestimates the flooded area for all the times shown. The two buildings located adjacent to the main channel bank (marked with A) cause higher water levels upstream of the buildings to be predicted by this model than those by FIVFLOOD, inducing a kind of backwater effect. Conversely, downstream of these buildings, lower levels are predicted both in the main channel and in the valley. The actual flooded volume is overall quite similar, but the inundated area is less. More correct is the prediction given by MCEP. Negligible differences exist on the inundation shape, while the area is almost the same.

The third test case is the flooding by levee failure. Figs. 10, 11, and 12 compare the results among the three models at the times 1,000 s and 2,000 s. As can be seen, the prediction obtained by the PA-31 model is less accurate at the beginning of flooding ($t=1{,}000$ s) than later. This is mostly due to the development of a strong shock wave that is not reproduced well. At later times a greater flooding occurs, and the accuracy improves ($t=2{,}000$ s). While differences still exist in the predicted depths, the general shape of the flooded area is, nevertheless, reproduced, and depths do not differ more than a few centimeters. The inundation predicted by MCEP has an entirely different shape and a lesser area. Here, the approximations derived by its conceptual nature are clearly evident due to the relevance of the dynamical terms.

Conclusions

A comparison between three 2D models for the simulation of flooding has been conducted. The models have different degrees of abstraction and can be assumed as representative of three different classes of models. The comparison was made only with respect to flooded areas—the most important result when mapping risk prone areas.

The comparison shows that neglecting dynamical terms, which underlies simplified modeling, can be questionable. Better results are generally obtained with the PA-31 model that more closely follow the actual dynamics of the process than with the MCEP model, which is more conceptual. The exception is the urban test case (case 2) in which MCEP better predicts the inundation area.

It appears that the dynamical terms should not be neglected in at least two situations:

1. When an irregular topography induces multiple paths in the flow, with abrupt turns, such as in the urban flooding example; and
2. When the shallow flow hypothesis cannot be assumed, as in the presence of shock waves or bores, such as in the levee breach example.

In these cases, the errors induced by a simplified model may be so large as to compromise any usefulness of the results.

For the purpose of assessing which areas will be inundated by a flood, the use of simplified models can give errors up to approximately 10%, while offering on the other hand some savings in computational effort. It should be noted that this accuracy assessment assumes a perfect knowledge of the flooded watershed. Actual uncertainties in the available data and the difficult estimation of the model parameters may give rise to larger inaccuracies, depending on the model selection. Simplified models may then be a reasonable choice by requiring less information.

Acknowledgments

Part of the financial support for this research was funded by the C.N.R.-G.N.D.C.I., Unità Operativa 1.30 di Napoli, whom the writers wish to thank. The writers also thank the reviewers for their help in improving the paper.

Notation

The following symbols are used in this paper:

g = gravity acceleration;
h = water depth;
m = efflux coefficient;
S_{fx} = friction term acting in x direction;
S_{fy} = friction term acting in y direction;
S_{0x} = bottom slope in x direction;
S_{0y} = bottom slope in y direction;
t = time;
u = velocity component in x direction;
v = velocity component in y direction; and
x,y = space coordinates.

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