Development of Modified Einstein Bed-load Equation for Sandy Stream in Malaysia

CHANG CHUN KIAT, *Science Officer, River Engineering and Urban Drainage Research Centre (REDAC), Universiti Sains Malaysia, Engineering Campus, Seri Ampangan, 14300 Nibong Tebal, Penang, Malaysia. Email: redac10@eng.usm.my*

AMINUDDIN AB. GHANI, *Deputy Director, REDAC, Universiti Sains Malaysia, Engineering Campus, Seri Ampangan,14300 Nibong Tebal, Penang, Malaysia. Email: redac02@eng.usm.my*

LIM HOOI WEN, *Research Student, School of Civil Engineering, Universiti Sains Malaysia, Engineering Campus, Seri Ampangan, 14300 Nibong Tebal, Penang, Malaysia. Email: lim_charis@yahoo.com*

ABSTRACT

River is a foundation in building and expanding a civilization. It brings advantages to human and also be a source of disaster such as flood, soil erosion. River frequently adjusts its cross-section, longitudinal profile, course of flow and pattern through the processes of sediment transport, scour and deposition. Therefore, understanding and predicting the behavior of river is important to avoid disasters. Besides that, it is essential to understand principles of sediment transport for application to the solution of engineering and environmental problems associated with natural events and human activities. From the previous studies, the analysis for a total of 122 sets of data obtained from Kinta River Catchment (Ab. Ghani et al., 2003), 40 sets of data obtain from Pari River and Kerayong River (Yahaya, 1999) and 165 sets obtained from Langat River Catchment (Ariffin, 2003) for four commonly used bed load equations namely Einstein (1942), Einstein-Brown (1950), Meyer-Peter-Muller (1946) and Shields (1936) Equations had been carried out. These sediment equation assessments which based on average size of sediment (d50) showed that the equations unable to predict bed load transport accurately. Both equations are over-predicted as compared the measured values and it is confirmed that none of the existing bed load equations gave satisfactory performance when tested on local river data. Therefore, this study had employes local sediment transport data (Ab. Ghani et al., 2003, Yahaya, 1999 and Ariffin, 2003) to yield a better equation that can accurately predict bed load transport in Malaysian rivers. Verification had also been carried out using new set of sediment transport data collected at Kulim River (Chang, 2006). The result from the study shows that the recommended Modified Einstein Equation has yield an accuracy of 65% in predicting bed load transport for all the measured data.

Keywords: Sediment transport; sand-bed stream; Einstein bed load function, bed load.

1 Introduction

River is a dynamic system governed by hydraulic and sediment transport process. Over time, the river responses by changing in channel cross section, increased or decreased sediment carrying capacity, erosion and deposition along the channel, which affect bank stability and river morphology changes. In order to sustain cultural and economic developments along a river, it is essential to understand principles of sediment transport for application to the solution of engineering and environmental problems associated with its natural state and human activities. Nowadays, there are various sediment transport equations developed based on different approaches to predict bed load transport rates, and Einstein Bed Load Function is one of the developed based on probability approach which practically, being presented in the every major textbook on alluvial-river mechanics and sediment transport (Graf, 1971; Chang, 1988; Yang, 1991; Chien and Wan, 1999). This paper summarizes the recent results in this field based on field data collected at four river catchment in Malaysia i.e. Kinta, Kerayong, Langat and Kulim rivers catchment (Ab. Ghani et al., 2003, Yahaya, 1999 and Ariffin, 2003, Chang 2006). The objectives of the studies are as follow:

(a) To study the relationship of transport parameter (ϕ) and flow parameter (ψ) and values of parameters in Einstein Equation.
(b) To modify Einstein Bed Load Function by employing local sediment transport data to yield a better equation that can accurately predict bed load transport in Malaysian rivers.

2 Project Site

The data collection programme was implemented at four river catchments (Figure 1) in Malaysia from 1998 until 2006. Initially the study was carried out at Pari River at Taman Merdeka and Kerayong River at Kuala Lumpur in 1998 to 1999 (Yahaya, 1999). The second study was done at Kinta River Catchment which consists of four rivers, namely Kinta River, Raia River, Pari River and Kampar River (Ab. Ghani et al., 2003). Third study was also carried out at Langat River Catchment which consists of Langat River, Lui River and Semenyih River from 2000 until 2002 (Ariffin, 2004). The fourth study was later completed at Kulim River in 2006 (Chang, 2006).

Figure 1 Location of Rivers for the Present Study

3 Sediment Data Collection

Field measurements were obtained along the selected cross section at the study sites by referring to Hydrological Procedure (DID, 1976; DID, 1977). The data collection including flow discharge, bed load, water-surface slope and bed material were carried out and details of data collection and analysis are given in Ab. Ghani et al. (2003), Chang et al. (2004) and Chang (2006).

3.1 Flow Discharge

A range of flow discharge measurements covering low and high regime were carried out using current meter. The procedure of flow discharge measurement was based on Hydrology Procedure No. 15: River Discharge Measurement by Current Meter (DID, 1976). Measurements taken include flow depth (y_o), velocity (V), and water surface width (B).

3.2 Bed Load

Bed load samples have been collected using Helley-Smith sampler at seven measuring points for each cross section. The bed load transport rate (T_b) was computed based on these seven samples. Details of computation of bed load measurement are given in Chang et al. (2004).

3.3 Bed Material

River bed materials were collected using Van Veen grab sampler. Seven samples were collected at points similar to those of bed load sampling. An average sediment size (d_{50}) was used for the present study analysis. A total of 349 data sets were obtained at the four study sites. Table 1 shows a summary with ranges for discharge (Q), water-surface width (B), flow depth (y_o), hydraulic radius (R), water-surface slope (S_o), mean sediment size (d_{50}), aspect ratio (B/y_o) and bed load (T_b). The mean sediment sizes for all sites showed that the study reaches are sand-bed streams where d_{50} ranges from 0.40 to 3.0 mm. The aspect ratios for the study sites at four river catchments are between 11 and 107 indicating that they are moderate-size channels. The water-surface slopes of the study reaches were determined by taking measurements of water levels over a distance of 200 m where the cross section is located (FISRWG, 2001). For all study sites the water-surface slopes were found to be mild with ranges between 0.001 and 0.005.

3 Sediment Transport Equations Assessment

The evaluation for a total of 349 data set based on average size of sediment (d_{50}) have been obtained for four commonly used bed load equations namely Einstein Bed Load Function (1942, 1950), Einstein-Brown Equation (1950), Meyer-Peter-Muller Equation (1948) and Shields Equation (1936). The performances of the equations were measured using the discrepancy ratio values, which is the ratio of the predicted load to measured load. In this study, a discrepancy ratio of 0.5 to 2.0 was used in the evaluation of sediment assessment.

From the evaluation of four bed load equations, it showed that all of the bed load equation gave unsatisfactory performance to predict the sediment load (Table 2) and the equations are over-predicted as compared the measured values. Both four equations have produced an average discrepancy ratio greater than 10. Therefore, it is confirmed that none of the existing bed load equations gave satisfactory performance when tested on local river data.

4 Modification of Einstein Bed-load Equation

Most of the bed-load equations such as Einstein bed load function, Einstein-Brown Equation, Meyer-Peter-Muller Equation, as well as total load equations such as Engelund & Hansen Equation (1967) and Graf (1971) Equations employ the transport parameter (ϕ) and flow parameter (ψ). The general relationship between these two parameter is given as below:

$$\phi = f(\psi) \qquad (1)$$

The Einstein bed load function was determined based on empirical data, which expressed the ϕ and ψ relationship as

$$A\phi = f(B\psi) \qquad (4)$$

Where A and B are constants incorporating awkward details about particle shape and step length, as well as water velocity distribution. The transport parameter (ϕ) and flow parameter (ψ) can define as:

$$\Psi = \frac{(S_s - 1)d_{50}}{RS_o} \qquad (2)$$

$$\phi = \frac{C_v VR}{\sqrt{g(S_s - 1)d_{50}^{\,3}}} \qquad (3)$$

S_s is the specific gravity of sediment, R the hydraulic radius, S_o the water surface slope, C_v the volumetric concentration and g the gravity acceleration.

The corresponding value of ϕ, is determine using empirically derived graph provided by Einstein (1950) as shown in Figure

2. However, Einstein could not deride the exact form of the relationship between ϕ and ψ because too many variables were unknown (Ettema & Mutel, 2004). As mentioned in Section 3, the evaluation for a total of 342 data set based on average size of sediment (d_{50}) have been obtained for Einstein Bed Load Function. Example of the predicted bed-load against measured bed-load using Einstein function is shown in Figure 3. However, it is found that Einstein Function had over-predicted the bed-load as compared the measured bed-load and none of the data produce a discrepancy ratio of 0.5 to 2.0.

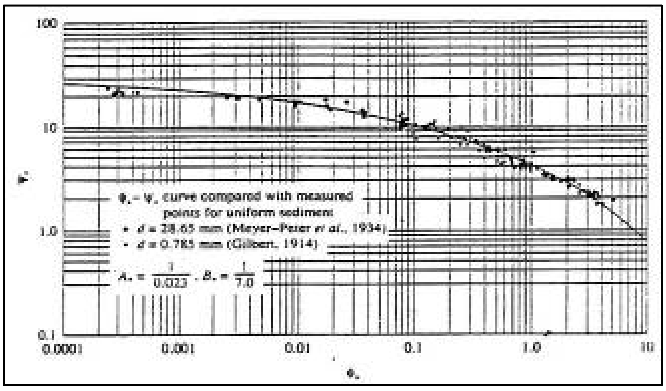

Figure 2 Einstein Bed-load Function (1950)

Table 1 Range of Field Data for Present Study (Yahaya, 1999; Ab. Ghani et al., 2003; Ariffin., 2003; Chang, 2006)

Study Site	Kinta River Catchment						
Location	Kampar River @ KM 34	Raia River @ Kampung Tanjung	Raia River @ Batu Gajah	Kinta River @ Ipoh	Pari River @ Manjoi	Pari River @ Buntong	Pari River @ Taman Merdeka
No. of Sample	21	20	21	20	20	20	16
Discharge, Q (m^3/s)	7.98 - 17.94	3.60 - 8.46	4.44 - 17.44	3.80 - 9.65	9.72 - 47.90	9.66 - 17.04	5.28 – 24.35
Water surface width, B (m)	20.2-21.1	22.2-25.6	17.3-20.8	24.6 - 28.0	20.3	19.3 - 19.5	18.0
Flow depth, y_o (m)	0.55-1.28	0.24-0.49	0.41-1.76	0.35 - 0.57	0.69-1.87	0.68 - 0.89	0.54 – 1.30
Hydraulic radius, R (m)	0.52-1.14	0.23-0.47	0.39-1.51	0.31 - 0.55	0.65-1.77	0.63 - 0.81	0.51 – 1.13
Water surface slope, S_o	0.0010	0.0036	0.0017	0.0011	0.0011	0.0012	0.0013
Mean sediment size, d_{50} (mm)	0.85 - 1.10	0.60 -1.60	0.50 - 0.85	0.40 - 1.00	1.70 - 3.00	0.85 - 1.20	2.00 – 3.10
B/y_o	17 – 38	46 – 107	12 – 45	48 – 86	11 – 29	22 – 29	14 - 34
Bed load, T_b (kg/s)	0.40 - 1.25	0.20 - 1.82	0.25 - 1.37	0.02 - 1.21	0.40 - 0.80	0.35 - 0.79	0.31 – 0.75
Study Site	Kerayong River Catchment	Langat River Catchment				Kulim River Catchment	
Location	Kerayong River @ Kuala Lumpur	Langat River @ Kajang	Langat River @ Dengkil	Lui River @ Kg Lui	Semenyih River @ Kg Sg Rinching	Kulim River @ CH 14390	Kulim River @ CH 3014
No. of Sample	24	20	3	92	50	10	12
Discharge, Q (m^3/s)	0.85 – 6.08	3.75 – 39.56	33.49 – 87.79	0.74 – 17.17	2.60 – 8.04	0.73 – 3.135	3.73-9.98
Water surface width, B (m)	18.0	15.0 - 20.0	30.0 - 33.0	15.0 – 17.0	13.5 – 15.0	9.0 – 13.0	13.0-19.0
Flow depth, y_o (m)	0.22 – 0.59	0.45 - 1.39	1.90 - 3.23	0.23 – 0.99	0.36 – 0.82	0.20 – 0.54	0.36-0.58
Hydraulic radius, R (m)	0.21 – 0.55	0.42 - 1.22	1.70 - 2.66	0.22 – 0.89	0.34 – 0.73	0.23 – 0.57	0.40-0.63
Water surface slope, S_o	0.0013	0.004 – 0.005	0.0167	0.0003 – 0.009	0.0023 – 0.015	0.001	0.001
Mean sediment size, d_{50} (mm)	2.80 – 3.00	0.37 – 2.13	0.52 – 0.95	0.50 – 1.74	0.88 – 2.29	1.00 – 2.40	1.10-2.00
B/y_o	30.5 – 81.82	14.4 – 33.5	9.30 – 17.4	17.2 – 65.8	17.1 – 41.5	23.4 – 44.8	26.0 - 52.5
Bed load, T_b (kg/s)	0.09 – 0.23	0.02 – 1.29	0.27 – 0.65	0.04 – 1.55	0.65 – 3.16	0.06 – 0.33	0.11-0.36

Table 2 Summary of Sediment Transport Assessment

Study Site	Location	Total of Data	Discrepancy Ratio (0.5 – 2.0)							
			Einstein		Einstein- Brown		Meyer-Peter- Muller		Shields	
			%	Average	%	Average	%	Average	%	Average
Kinta River Catchment (Ab. Ghani et al., 2003)	Pari River @ Manjoi	20	0	89.3	0	73.2	0	57.4	0	230.1
	Pari River @ Buntong	20	0	57.0	0	77.0	0	38.8	0	231.6
	Raia River @ Kampung Tanjung	20	0	76.9	0	265.8	0	65.4	0	387.5
	Raia River @ Batu Gajah	21	0	33.3	0	411.8	0	48.7	0	396.2
	Kinta River @ Ipoh	20	0	78.5	0	96.0	0	54.4	0	216.4
	Kampar River @ KM 34	21	0	27.5	0	33.2	0	18.3	0	88.84
Kinta River Catchment (Yahaya, 1999)	Pari River @ Taman Merdeka	16	0	36.5	0	24.0	0	27.6	0	80.7
Kerayong River Catchment (Yahaya, 1999)	Kerayong River @ Kuala Lumpur	24	0	23.8	4.2	9.1	4.2	20.7	4.2	24.0
Langat River Catchment (Ariffin, 2003)	Langat River @ Kajang	20	0	55.2	0	19112.1	0	1216.7	0	14016.9
	Lui River @ Kg Lui	92	0	64.3	0	1838.6	0	158.0	0	948.0
	Semenyih River @ Kampung Sungai Rinching	50	0	23.3	0	22.61.1	0	92.0	2	923.0
Kulim River Catchment (Chang, 2006)	Kulim River @ CH 14390	12	0	15.5	0	6.3	0	11.28	0	18.2
	Kulim River @ CH 3014	10	0	34.6	0	19.74	0	24.8	0	81.2

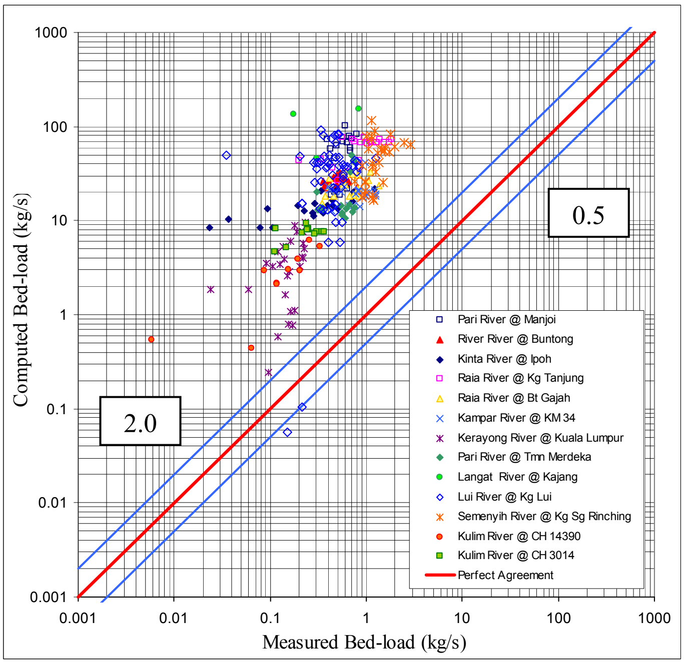

Figure 2 Predicted Bed-Load against Measured Bed-Load Using Einstein Function (1942)

Brown (1950) developed a bed load equation based on Einstein's (1942) Equation with the relationship between ϕ and ψ, i.e.

$$A\phi = e^{-B\psi} \qquad (5)$$

Where, A = 0.465 and B = 0.391, $\psi > 5.5$, $\phi > 0.4$

and

$$\phi = 40\left(\frac{1}{\psi}\right)^3 \qquad (6)$$

Where, $\psi \leq 5.5$

Since the sand-bed streams (d_{50} ranges from 0.40 to 3.0 mm) in the present study are moderate-size channels with an aspect ratio between 11 and 107 and of mild slope (0.001 – 0.005), average discrepancy ratio are in 20-100 range, compare to the evaluations using Einstein-Brown, Meyer-Peter-Muller and Shields equations, attempts were made to modify Einstein equation for predicting bed-load transport by employing the transport parameter (ϕ) and flow parameter (ψ). The development of the Modified-Einstein Equation based on the 342 sets of field data with performing the outlier was carried out by modifying the A value and B value which are constant values as shown in Equation 5, the following equation had been produced,

$$3.811\ \phi = e^{-0.491\psi} \qquad (7)$$

Therefore, the computed bed load (T_b) is given in equations 8 and 9 as follow:

$$C_v = \frac{Q_b}{Q} \qquad (8)$$

and $$T_b = Q_b \times \rho_s \qquad (9)$$

Q_b is bed load discharge and ρ_s is density of natural sediments, which is approximately equal to 2650 kg/m^3 (Soulsby, 1997).

Table 3 gives a summary of accuracy for Equations 7 based on the discrepancy ratio (0.5 – 2.0) for the total of 342 data and Figure 3 plots the transport parameter (ϕ) against flow parameter (ψ) using Equation 7. From the analysis, the recommended Equation 7 has yield an accuracy of 65% in predicting bed load transport for all the measured data. The average discrepancy ratio of Equation 7 for all 342 river data is 1.68. This means that, Equation 7 suggesting the viability of using this new modified equation for predicting flow discharge for the rivers with similar characteristics as studied. Figure 4 plot predicted bed-load against measured bed-load using the Modified Einstein Equation.

Table 3 Application of Modified Einstein Equation in Rivers Data of Malaysia

Study Site	Location	Total of Data	Discrepancy Ratio (0.5 – 2.0)		
			No. of data	%	Average
Kinta River Catchment (Ab. Ghani et al., 2003)	Pari River @ Manjoi	20	8	40.00	2.54
	Pari River @ Buntong	20	18	90.00	1.59
	Raia River @ Kampung Tanjung	20	16	80.00	1.84
	Raia River @ Batu Gajah	21	20	95.23	0.95
	Kinta River @ Ipoh	20	14	70.00	2.38
	Kampar River @ KM 34	21	18	85.71	0.79
Kinta River Catchment (Yahaya, 1999)	Pari River @ Taman Merdeka	16	11	68.75	1.35
Kerayong River Catchment (Yahaya, 1999)	Kerayong River @ Kuala Lumpur	24	12	50.00	0.51
Langat River Catchment (Ariffin, 2003)	Langat River @ Kajang	16	7	43.75	2.89
	Lui River @ Kg Lui	92	42	45.65	2.40
	Semenyih River @ Kampung Sungai Rinching	50	40	80.00	1.06
Kulim River Catchment (Chang, 2006)	Kulim River @ CH 14390	10	4	40.00	0.41
	Kulim River @ CH 3014	12	12	100.00	1.17
Total of Data		342	222	64.91	1.68

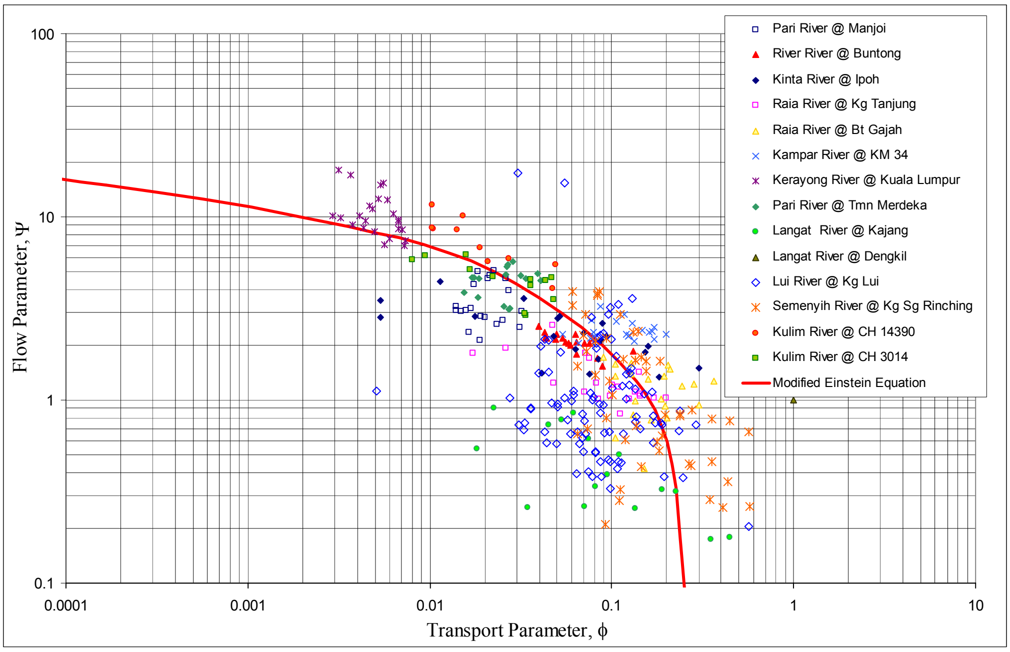

Figure 3 Flow Parameter (ψ) against Transport Parameter (ϕ)

5 Conclusions

Applications of existing bed load equations yielded bed-load transport over-predicted as compared the measured values and it is confirmed that none of the existing bed load equations gave satisfactory performance when tested on local river data. Using the recommended Modified-Einstein Equation, the computed bed load transport rates were in much closer agreement with the actual measured values for application to the moderate-size and sand-bed streams in Malaysia. Based on 342 data collected from Kinta, Kerayong, Langat and Kulim river catchments (performing the outlier), the present study also indicates that employing local sediment transport data yielded a better equation that can predict bed load transport in Malaysian rivers more accurately.

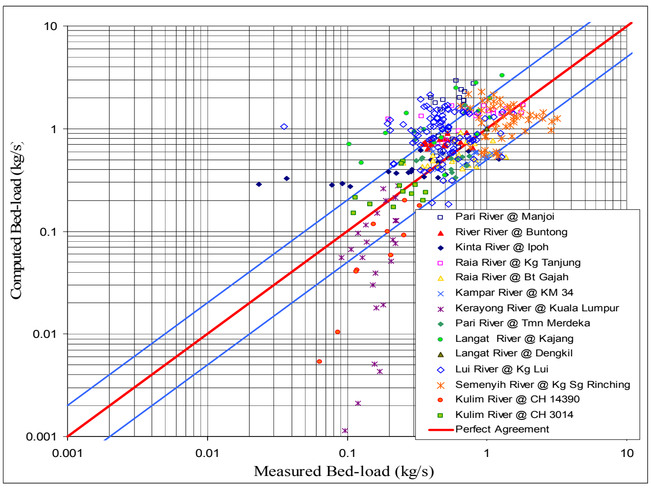

Figure 4 Predicted Bed-Load against Measured Bed-Load Using Modified Einstein Equation

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