Radial Basis Function to Predict Bridge Pier Scour


ABSTRACT

Bridge pier scouring is a significant problem in the safety estimation of bridges. Extensive experimental studies have been conducted on the prediction of local scour depth at bridge piers. Kafi and Alam (1995) found out that the accuracy of the laboratory based equations can be improved if the coefficients and exponents of these equations were derived using field data. Ab. Ghani and Nalluri (1996) extended the study by Kafi and Alam (1995) and derived equations to further improve the prediction of local scour depth at bridge piers using the statistical method such as the regression technique. The traditional statistical analysis is generally replaced in many fields of engineering by the alternative approaches of artificial neural networks (ANNs). This paper presents an alternative to the regression in the form of ANNs. It has been found that ANN performs well as compared to the regression equation (HEC-18).

INTRODUCTION

The presence of bridge pier causes an abrupt change in the direction of approach flow resulting in the removal of bed material hence local scout at piers. The mechanism of flow around a pier structure is so complicated that it is difficult to establish a general empirical model to provide accurate estimation for scour. Interestingly, each of the proposed empirical formula yields good results for a particular data set (Bateni et al. (2007). The failure of Black Mount Bridge of New Zealand was caused by the undermining of its piers in the riverbed (Melville and Coleman 2000). To avoid such failures, the pier foundation is to be constructed to a depth deeper than the maximum possible scour depth in its lifetime. Hence a reliable estimate of the maximum possible scour depth around bridge pier is of paramount importance in safety and economic design of foundation of bridge piers.

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Local scour problem around a pier

Local scour around bridge piers and abutments has remained a major cause of bridge failures induced by hydraulic deficiencies. Combined effects of turbulent boundary layer, time-dependent flow pattern, and sediment transport mechanism in and around the scour hole make the phenomenon extremely complex (Yanmaz and Ustun, 2001). Equilibrium scour depth around a circular pier in a steady flow over a bed of uniform, spherical and cohesion-less sediment depends on numerous groups of variables such as flow, sediment characters, and pier geometry. The basic similitude requirements for hydraulically modeling the simplest of pier-scour situations are difficult to satisfy. For example, studies employed laboratory flumes, which were different from natural channels that are non-rectangular with rough and mobile banks (Mohammed et al., 2005). Local scour at piers is a function of many variables. Scour depth at a pier as in Fig. 1, depends on variables characterizing the fluid, flow, bed sediment and pier. Thus, the following functional relationship can describe scour depth (Ettema et al., 1998):

![Diagram of flow and local scour around a circular pier](image)

Fig. 1: Flow and local scour around a circular pier (Bateni et al. 2007).

![Diagram of RBF Neural Network Architecture](image)

Fig. 2: RBF Neural Network Architecture
\[ d_{se} = f(\rho, \mu, U, Y, g, d_{50}, U_c, D) \] (1)

in which \( \rho \) is the fluid density; \( \mu \) the fluid dynamic viscosity; \( U \) the average velocity of approach flow; \( Y \) the flow depth; \( g \) the gravitational acceleration; \( d_{50} \) the particle mean diameter; \( U_c \) the critical value of \( U \) associated with initiation of motion for particles on bed surface; \( D \) the diameter of the pier and \( d_{se} \) the equilibrium scour depth. The eight independent variables in (1) are reducible to a set of five non-dimensional parameters. If \( \rho, U, \) and \( D \) are chosen as repeating variables, the following functional relationship describes scour depth normalized with pier diameter:

\[ \frac{d_{se}}{D} = \psi \left( \frac{U}{U_c}, \frac{U}{\sqrt{gY}}, \frac{Y}{D}, \frac{d_{50}}{\mu}, \frac{\rho UD}{\mu} \right) \] (2)

Several approaches have been presented to estimate equilibrium scour depth. A summary of these approaches is shown in Table 1. Johnson (1995) reported that Melville and Sutherland formula tend to over-predict the depth of local scour to a greater extent than any of other formulae. Recently, Mohammed et al (2005) showed that the Laursen and Touch and the Colorado State University (CSU) formulae give reasonable estimations, whilst the Melville and Sutherland and Jain and Fisher formulae over-predict the depth of scour based on the comparison of some bridge pier scour formulae using field and laboratory data. So the same parameters are considered for ANN model inputs (\( b/y \), and \( Fr=V/(gy)^{1/2} \)) and output (\( d_{se}/y \)) in this study.

**Table 1: The Existing Bridge Pier Scour Equations**

<table>
<thead>
<tr>
<th>Researcher</th>
<th>Equation</th>
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<tbody>
<tr>
<td>Ab. Ghani and Nalluri (1999)</td>
<td>[ \frac{D_s}{d} = \beta_0 \left( \frac{v_o}{b} \right)^{\beta_1} \left( \frac{b}{d} \right)^{\beta_2} \left( \frac{Q}{by_o \sqrt{gy_o}} \right)^{\beta_3} ]</td>
</tr>
<tr>
<td>Laursen and Toch (1956)</td>
<td>[ \frac{D_s}{b} = \beta_0 K \left( \frac{v_o}{b} \right)^{\beta_1} ]</td>
</tr>
<tr>
<td>Shen (1971)</td>
<td>[ D_s = \beta_0 (Re)^{\beta_1} ]</td>
</tr>
<tr>
<td>Hancu (1971)</td>
<td>[ \frac{D_s}{b} = \beta_0 \left( \frac{v^2}{gb} \right)^{\beta_1} ]</td>
</tr>
<tr>
<td>Qadar (1994)</td>
<td>[ D_s = \beta_0 C_o^{\beta_1} ]</td>
</tr>
<tr>
<td>HEC -18 (1993)</td>
<td>[ D_s = 2.1 \left( \frac{b}{y} \right)^{0.65} Fr^{0.43} ]</td>
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</table>

Most of the scour depth prediction formulae available in the literature have been developed based on the analysis of the scour parameters using the statistical method such as the regression
method. Conventional statistical analysis is generally replaced in many fields of engineering by the alternative approaches like artificial neural networks (ANNs) and adaptive neuro-fuzzy inference system (ANFIS) (Bateni et al., 2007). The relatively recent technique of ANN has been reported to provide better solutions for hydraulic engineering problems, in cases where the available data is incomplete or ambiguous by nature (Azamathulla et al., 2005).

**Neural Network Model**

As known widely by now, neural networks provide random mapping between an input and an output vector by mimicking the biological cognition process of our brain. A typical network would consist of three layers of neurons, namely, input, hidden and output, with each neuron acting as an independent computational element. Neural networks derive their strengths from a ‘model-free’ processing of data and a high degree of freedom associated with their architecture. Details of concepts involved in neural networks along with their applications in water resources can be seen in The ASCE Task Committee (2000), Dawson and Wilby (2001) and Maier and Dandy (2000). The text book of Kosko (1992) give details of the network theory. Typical problems tackled by the networks in hydraulic engineering include, estimation of pier scour (Trent et al. 1999), prediction of flow conditions when the interfacial mixing in stratified estuaries commences Grubert (1995), prediction of the scour depth at culvert outlets Liriano and Day (2001) and prediction of sediment load concentration in rivers using neural networks (Nagy et al., 2002).

Before actual application, the network has to be trained from examples. Training comprises presentation of input and output pairs to the network and fixing the values of connection weights, bias or centers. The training may require many epochs (presentation of complete data sets once to the network). Generally the network is presented with the input and output pairs till the training sum-square error reaches the error goal in order to give the desired network performance.

In the present study the usual feed forward type of network was considered. It was trained using radial basis function (RBF). Concepts involved behind these training schemes are outlined in the ASCE Task Committee (2000).

The neural network toolbox contained within the MATLAB software packages was utilised in this study. A survey of the literature reporting such observations indicated that only five types of information, namely, pier width b, flow depth y, bed material $d_{50}$, scour depth, $d_s$, and flow velocity, are uniformly reported in references (Landers an Mueller, 1999 and Mohammed et al., 2005). Table 2 summarises the ranges of field data available i.e. pier width (b), flow velocity (V), flow depth (y), mean diameter of bed material ($d_{50}$), and observed scour depth ($d_s$).

<table>
<thead>
<tr>
<th>Table 2: Ranges of field data</th>
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<tr>
<td><strong>Variables</strong></td>
</tr>
<tr>
<td>b (m)</td>
</tr>
<tr>
<td>V (m/s)</td>
</tr>
<tr>
<td>y (m)</td>
</tr>
<tr>
<td>$D$ (m)</td>
</tr>
<tr>
<td>$d_{50}$ (mm)</td>
</tr>
</tbody>
</table>

From the previous experience (Azamathulla et al., 2005), grouped variables produced good results, so in the present study inputs used were b/y and F, and output was $d_s/y$. 
Out of the total of 398 input-output pairs, 75 percent, selected randomly, were used for training and remaining 25 percent were employed for testing or validation. As dictated by the use of Gaussian function all patterns were normalized within the range of (0.0, 1.0) before being used. The RBF network various values of spread (σ) between 0 and 1 were tried out. The spread value of 0.01, which yielded the best performance on both training and testing data, was selected.

The use of a soft computing tool like artificial neural networks (ANNs) in place of the regression for the problem under consideration met with large success as shown in hydraulic engineering (Azamathulla et al., 2005 and 2006). This inspired the present work in which the scour problem is tackled with the help of another soft computing tool namely, RBF.

The majority of previous works on scour predictions are based on hydraulic model studies (Bateni et al. 2007). While hydraulic model studies have advantages such as repeatability, they have helped more in exploring the scour mechanism than in obtaining more accuracy in the depth estimation. Scale effects, inability to correctly model certain field conditions like bed morphology and loss of flow energy in aeration, and failure to consider a variety of causative factors simultaneously are some of the deficiencies associated with the model measurements. It was, thus, decided to calibrate the neural networks (RBF) with the help of realistic field conditions only, although it is recognized that prototype measurements may also suffer from instrumental uncertainties and inaccuracies and lack of availability of data on all causative parameters.

**RBF Model Development and Validation**

From the 398 data sets used in this study, 300 data sets (around seventy five percent of these patterns, chosen randomly until the best training performance was seen) were used for training while remaining ones were used for testing or validating RBF model.

**Analysis and Results**

Figures 3 and 4 show the outcome in the form of scatter plots for the testing set of data, not involved in calibrating the RBF and HEC-18. For use in practice the predictions are given in terms of non-dimensional scour depth in these figures. The non-dimensionalization was done by dividing these quantities by corresponding flow water depths. An excellent prediction made by the RBF can be seen.

The HEC-18 formula, suggested by the FHWA (1993) showed a large error margin with respect to the neural network (RBF). This is quantitatively reflected in the error statistics (Appendix I) of the correlation coefficient, R, average error AE, the average absolute deviation, δ and the root-mean-square error, RMSE (Table 3). A look into all of their scatter plots and tables of error statistics revealed that the prediction accuracies of the present RBF model are generally comparable to the HEC-18. For engineering applications the average percentage errors reflected in the statistic, ‘ä’ could be more attractive, indicating better acceptability of the RBF. However the results seem to

<table>
<thead>
<tr>
<th>Figure No.</th>
<th>Method</th>
<th>Correlation co-efficient, R</th>
<th>Av.error,AE</th>
<th>Av. abs.deviation,d</th>
<th>RMSE</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>RBF</td>
<td>0.96510</td>
<td>-1.73136</td>
<td>12.1501</td>
<td>0.042331</td>
</tr>
<tr>
<td>4</td>
<td>HEC18</td>
<td>0.62729</td>
<td>-17.45</td>
<td>11.49</td>
<td>0.298756</td>
</tr>
</tbody>
</table>
be not as certain in high-value predictions, as reflected in somewhat lower RMSE and $R$, which are the measures sensitive to errors at larger observations. It is also possible that the flexibility in the data mining approaches incorporated (ANNs) may have reached an acceptable level for the given sample size and hence very large variations in the accuracy levels by either method may not be possible to achieve.
CONCLUSIONS

The RBF model was developed to obtain the values of relative scour depth from the field measurements. The developed RBF model was found to produce scour values that were fairly accurate and comparable with the earlier works based on artificial neural networks, when multiple error criteria were applied using hydraulic model data (Bateni et al. 2007).

The present study once again highlighted the necessity to replace the traditional deterministic equations-based approaches by the soft schemes.

NOTATION

- \( d_e \) equilibrium scour depth
- \( b \) pier width
- \( g \) gravitational acceleration
- \( n \) Number of observations
- \( q \) discharge intensity
- \( y \) flow depth
- \( K \) constant depending on pier shape
- \( d_{s0} \) particle mean diameter
- \( R \) Correlation coefficient
- \( U \) the average velocity of approach flow
- \( V \) flow velocity
- \( Fr \) Froude number
- \( AE \) Average error
- \( RMSE \) Root Mean Square Error
- \( \delta \) Average Absolute Deviation
- \( \rho \) fluid density
- \( \mu \) fluid dynamic viscosity;
- \( U_c \) the critical value of \( U \) associated with initiation of motion of particles on bed surface;

REFERENCES


APPENDIX I. THE ERROR MEASURES

Correlation coefficient (R),

\[
R = \frac{\sum xy}{\sqrt{\sum x^2 \sum y^2}}
\]  

(3)

Where \( x = (X - \overline{X}) \), \( y = (Y - \overline{Y}) \), \( X = \) Observed values, \( \overline{X} = \) Mean of \( X \), \( Y = \) Predicted value, \( \overline{Y} = \) Mean of \( Y \). The summation in the above equation as well as in the following two equations is carried out over all ‘n’ number of testing patterns.

Average error (AE),

\[
AE = \frac{\sum \frac{X - Y}{X} \times 100}{n}
\]

(4)

Root Mean square error (RMSE),

\[
RMSE = \left( \frac{\sum (X - Y)^2}{n} \right)^{1/2}
\]

(8)

Average absolute deviation, \( \delta \):

\[
\delta = \frac{\sum |(Y - X)|}{\sum X} \times 100
\]

(5)