Linear Programming Approach for Irrigation Scheduling – A case Study

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Abstract
There is an increasing awareness among irrigation planners and engineers to design and operate reservoir systems for maximum efficiency to maximize their benefits. Accordingly, significant work has been done on reservoir operation for known total irrigation demand and on the optimal allocation of water available to crops at the farm level.Very few studies have been conducted to derive optimal reservoir operation policies integrating the reservoir operation with the on-farm utilisation of water by the various crops. This present paper deals with the development of model -- Linear Programming (LP) -- to be applied to real-time reservoir operation in an existing Chiller reservoir system in Madhya Pradesh, India.

Keywords: Cropping pattern, Water resource management, Irrigation management, Optimization

1. Introduction

In most developing countries, a huge share of the limited budget goes to creating facilities for irrigation. Construction of reservoirs requires very high investment and also causes socioeconomic and environmental issues. Water in the reservoir has multiple claimants and needs to be optimally utilized to generate maximum benefits through proper operation, which must remain consistent despite uncertain future inflows and demands. According to the World Commission on Dams, many large storage projects worldwide are failing to produce the anticipated benefits (Labadie, 2004). Similarly, small storage projects made for local areas in developing countries, like India, are also failing to meet expectations. The main cause identified at various levels of discussion, as reported by Labadie (2004), is inadequate consideration of the more mundane operation and maintenance issues once the project is completed. For existing reservoirs, optimum operation is critical, since all the expected benefits are based on timely water releases to meet the stipulated demand. Real-time operation of a reservoir requires making relatively quick decisions regarding releases based on short-term information. Decisions are dependant on the storage in the reservoir and information available in the form of forecast hydrologic and meteorological parameters. This is especially important during floods and power generation, where the system has to respond to changes very quickly and may need to adapt rapidly (Mohan et al. 1991). For reservoir systems operated for irrigation scheduling, real-time operation is not very common because of longer decision steps. Traditionally, the reservoirs meant for irrigation purposes are operated on heuristics and certain rules derived from previous experiences. This defies the concept of water-management; much of the water is lost, which in turn leads to loss of revenue.

In the early 1960s, mathematical programming techniques became popular for reservoir planning and operation; pertinent literature is available. An excellent review of the topic is given by Yeh (1985), followed by Labadie (2004) and Wurbs (1993). Along with simulation studies, Linear Programming (LP), Dynamic Programming (DP) and Non Linear Programming (NLP) are the most popular modelling techniques. A comparative study on the applicability and computational difficulties of these models is presented by Mujumdar and Narulkar (1993).

Many of the aforementioned techniques have been implemented in realistic scenarios, and many reservoir systems worldwide are operated based on the decision rules generated from these techniques. However, there exists a gap between theory and practice, and full implementation has not been achieved yet (Labadie, 2004).

The basic difficulty a reservoir manager faces is to take a real-time optimum decision regarding releases according to the future demand and inflow. This leads to the problem of optimization of the stochastic domain. Two approaches of stochastic optimization are practised: i) Explicit Stochastic Optimization (ESO), which works on probabilistic descriptors of random inputs directly and ii) Implicit Stochastic Optimization (ISO), which is based on historical, generated or forecasted values of the inputs through the use of Time Series Analysis or other Probabilistic approaches. The ESO approach has computational difficulties; ISO methods are simple, but require an additional forecasting model for real time operation.

In the case of irrigation reservoirs, decision making at the reservoir level depends upon the water demand arising at the field level. In order to operate the reservoir in the best possible way, it becomes imperative to understand the processes occurring in the crop-soil-water-atmosphere system. This helps not only in the estimation of accurate demands, but also ensures optimum utilisation of water. If the processes at the field level are also modelled properly and integrated with the reservoir level model, the goal of water management can be achieved in the best possible way. Dudley et al. (1971) pioneered the integration of the systems in the determination of optimal irrigation timing under limited water supply using a Stochastic DP model. Dudley and his associates then improved the model (Dudley and Burt, 1973; Dudley, 1988; Dudley and Musgrave, 1993). Vedula and Mujumdar (1992, 1993) and Vedula and Nagesh Kumar (1996) have also contributed to this area. Their approach was to derive a steady state reservoir operation policy while maximizing the annual crop yield. DP-SDP and LP-SDP were used in the modelling. However, for real-time reservoir operation, Vedula and Nagesh Kumar (1996) stressed the need to forecast inflows and rainfall in the current season to implement the steady state operation policy. As a result, the ESO model has to be supplemented with an ISO model to get a policy for the current period. As an extension to the work of Vedula and Mujumdar (1992), a significant contribution to the real-time reservoir approach was presented by Mujumdar and Ramesh (1997). They addressed the issue of short term real-time reservoir operation by forecasting the inflow for the current period, a crop production state variable and a soil moisture state variable. Their work was based on SDP, but had all the limitations of SDP regarding the curse of dimensionality.

Against this background, a model for the derivation of real-time optimal operating policy for a reservoir under a multiple crop scenario is proposed in the present study. The primary issue is that the reservoir gets inflows during the wet season (monsoon season) and is operated for irrigation in the dry season (non-monsoon season). The reservoir storage and the soil moisture level are considered to be the principal state variables, and the irrigation depths are the decision variables. An optimal allocation model is embedded in the integrated model to evaluate the irrigation water depth supplied to different crops whenever a competition for water exists amongst various crops. The model also serves as an irrigation-scheduling model because it specifies the amount of irrigation for any given fortnight. The impact on crop yield due to water deficits and the effect of soil moisture dynamics on crop water requirements are taken into account. Moreover, a root growth model is adopted to consider the effects of varying root depths on moisture transfer. The only stochastic element in the season is the evapotranspiration. The handling of stochasticity has been accomplished through dependability based forecasting in an ISO model. The rest of the variables, such as soil moisture status and the reservoir storage status, at the beginning of any period are considered to be state variables. The basic formulation is based on a LP model and is later transformed into a GA framework.

2. The Model Formulation and Concept

The real-time operation model proposed in the present study integrates the reservoir level and a field level decision (Figure 3). It considers the soil-moisture status and the reservoir storage as the state variables and the applied irrigation depths as decision variables. The formulation is based on the conceptual model for soil moisture accounting and the reservoir storage continuity relationships. A major emphasis is laid on maintaining soil moisture in a state such that the evapotranspiration from the crops takes place at a rate that achieves better results in the form of increased yields from the crops.

To assess the timing of irrigation water application, the soil moisture status of the crop is an important parameter. Whenever the soil moisture status approaches a critical limit, irrigation is applied. Thus, the soil moisture status is monitored either by physical measurement or through soil moisture models. Soil moisture models are more popular since they do not require a lot of instrumentation to be installed in the field. Soil moisture models can be formulated either by a physical approach (Fedders et al., 1978) or a conceptual approach (Rao, 1987). The conceptual approach has been used by Rao et al. (1988), Rao et al. (1990) and

Hajilal et al. (1998) for the problem of irrigation scheduling. Vedula and Mujumdar (1992) utilised the conceptual model in their study. The same concept is adopted in the present study.

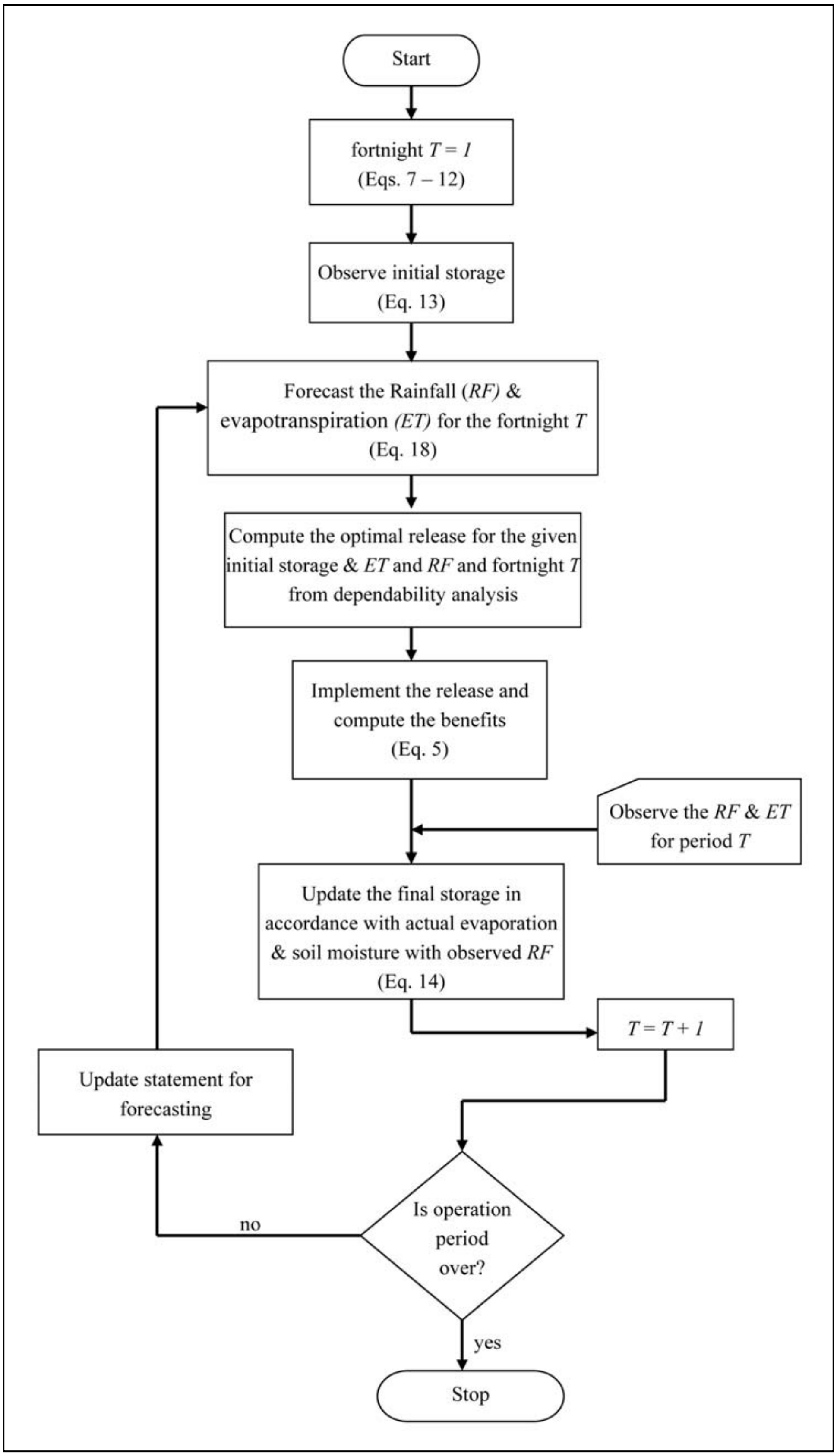

Figure 3 Flow chart of real-time operation of reservoir

3. The Conceptual Model

In the conceptual model for the Crop-Soil-Water-Atmosphere (CSWA) system, the basic assumption is that the soil acts as a reservoir, the main inputs to the reservoir are rainfall irrigation, and the main outputs are evapotranspiration, percolation and drainage. The extent of the reservoir is considered to be up to the effective root zone at the particular time. The soil water reservoir is governed by a continuity equation:

$$\theta_i^{k+1} ED_i^{k+1} - \theta_i^k ED_i^k - IRR_i^k + AET_i^k = RF^k \tag{1}$$

The conceptual model stated by Eq. 1 is used to compute the irrigation to be applied for the LP model with area as a decision variable. The following parameters are important for the conceptual model. Figure 1 shows the sketch for the conceptual reservoir. In the context of the conceptual model two parameters are important:

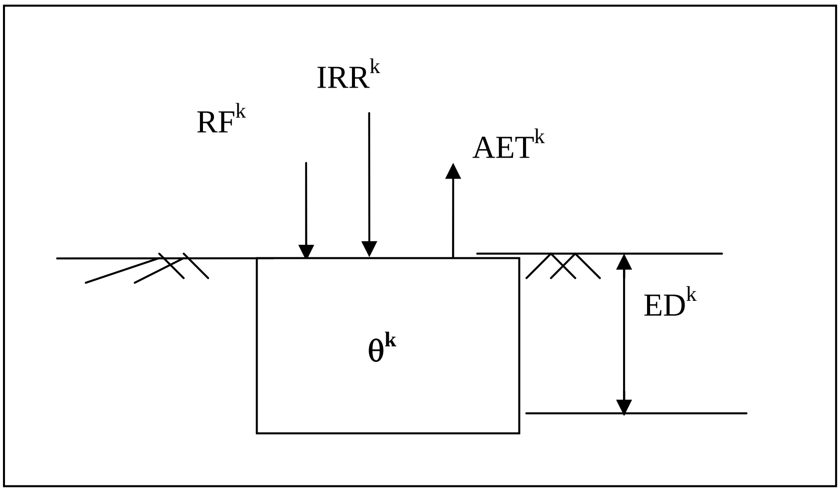

Figure 1 Conceptual model

Variation of Evapotranspiration with the Available Soil Moisture

Evapotranspiration as a function of the available soil moisture is expressed as:

$$AET_i^k = PET_i^k \quad \text{if} \quad a_{ai}^k \geq Zww \tag{2}$$

or

$$AET_i^k = \frac{a_{ai}^k}{Zww} PET_i^k \tag{3}$$

where AET_i^k is the actual evapotranspiration that has occurred from crop *i* in fortnight *k* (mm), PET_i^k is the potential evapotranspiration in a particular geographical location (mm), *Zww* is the critical available moisture limit (mm/cm) = (*Zf*–*Zw*) *d*, *Zf* is the field capacity for the soil (mm/cm), *Zw* is the permanent wilting point for the soil (mm/cm), d is the depletion factor and assumed to be 0.5 in the present study, and a_{ai}^k is the average available soil moisture over a fortnight (mm/cm). The average available soil moisture over a fortnight is given by

$$a_{ai}^k = \frac{a_i^k + a_i^{k+1}}{2.0}$$

where $a_i^k = \theta_i^k - Zw$ if $a_i^k < Zww$

otherwise $a_i^k = Zww$

A similar expression can be used for a_i^{k+1}.

Root Zone Depth Growth

The root depth data in relation to the time stages are prepared according to the Linear Root Growth Model (adopted by Narulkar, 1995). The model assumes that maximum root depth is achieved at the start of the yield formation stage. It remains at the maximum depth until the maturity stage. A minimum depth of 15 cm is considered in the first fortnight to account for the conditions of bare soil and an area with sparse crops. The root depth model is shown in Figure 2.

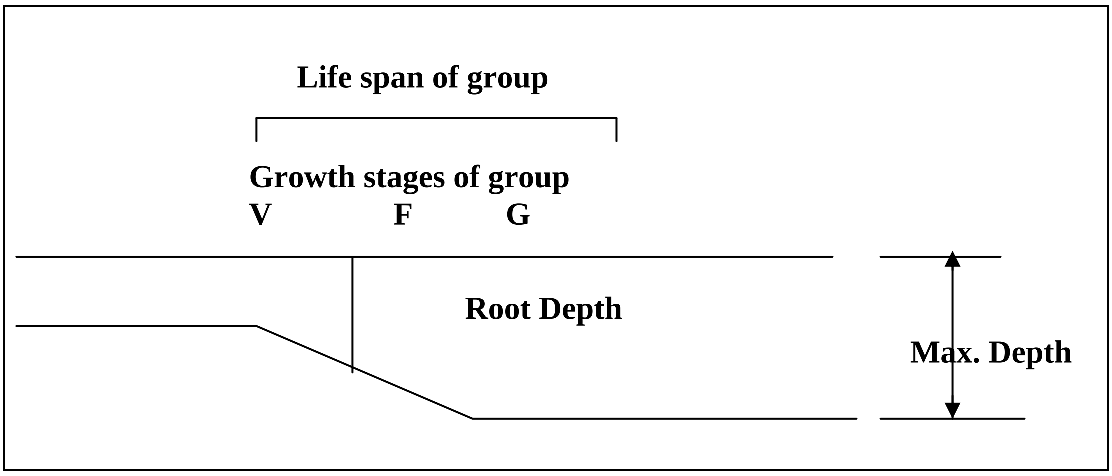

Figure 2 Root Depth growth model

Relative Yield Ratio

The yield of a crop is affected by water deficits and the rate of evapotranspiration. The rate of evapotranspiration tends to decrease depending on the available moisture content. There are many methods to model the phenomenon. However, the model used in the present study is the most commonly-adopted model. The relative yields are computed on the basis of the expression given by Doorenbos and Kassam (1979)

$$\frac{Y_{ai}}{Y_{mi}} = 1 - Ky^k \left(1 - \frac{AET_i^k}{PET_i^k}\right) \tag{4}$$

Equation (4) gives a yield ratio for a single period only. However, the aggregate effect of moisture deficits over all fortnights of crop growth is also evaluated. The final yield ratios computed for the crop during various time periods of a season is computed by a multiplicative model (Rao et al., 1990). The determination of the yield ratio is very important since they reflect the operation policy for an irrigation system. The expression is given by

$$\frac{Y_{ai}}{Y_{mi}} = \prod_{i=1}^{ncr} \left[1 - Ky^k \left(1 - \frac{AET_i^k}{PET_i^k}\right)\right] \tag{5}$$

Water Requirements of the Crops

The model derived for an optimal crop pattern uses predetermined irrigation demands. On the basis of this, the optimisation model selects an appropriate area for an individual crop. The irrigation demands are determined using the conceptual model stated in Eq. 1. The irrigation requirements may be calculated by substituting a value of critical soil moisture content instead of soil moisture in either of the fortnights *k* and *k+1* and replacing the values of actual evapotranspiration by potential evapotranspiration and rearranging the terms of Eq. 1:

$$IRR_i^k = \theta_{cr}\left(ED_i^{k+1} - ED_i^k\right) + PET_i^k \tag{6}$$

where θ_{cr} is the critical soil moisture content below which the actual evapotranspiration may fall below the potential rate.

4. Integrated LP Formulation

In the objective function, the weighted sum of all the actual evapotranspiration values is maximised. The weights are assigned according to the yield response factors for individual crops in individual periods. The objective is to maximise the actual evpotranspiration rate to minimise the deficits in the yields. The available soil moisture in any time period in the objective function is indirectly maximised:

$$MaxZ = \sum_{i=1}^{ncr}\sum_{k=1}^{np}\left\{\frac{a_i^k + a_i^{k+1}}{2.0}\right\}\frac{Ky^k}{Zww} \tag{7}$$

subject to the following constraints:

1. Soil moisture continuity

$$\theta_i^{k+1}ED_i^{k+1} - \theta_i^k ED_i^k - IRR_i^k + \left\{\frac{a_i^k + a_i^{k+1}}{2.0}\right\}\frac{PET}{Zww} = RF^k \tag{8}$$

where $$\theta_i^{k+1} - a_i^{k+1} - b_i^{k+1} = ZW \tag{9}$$

with physical bounds

$$\theta_i^{k+1} \le 4.0 \tag{10}$$

$$a_i^{k+1} \le 0.9 \tag{11}$$

2. Reservoir continuity

$$A^k S^{k+1} - B^k S^k + \sum_{i=1}^{ncr}\frac{IRR_i^k * AREA_i^k}{Eff} = -ID - A_o RE^k \tag{12}$$

$S^{k+1} \le 31.1$ (Maximum Reservoir Capacity M m^3) (13)

5. Crop Simulation Model

The optimisation model presented above yields some irrigation depth values that are based on forecasted values for the reference evapotranspiration. This reference evapotranspiration, in turn, is based on a dependability model. However, the actual evapotranspiration value differs from these values, and thus, before going into the next fortnight, the soil moisture status must be updated with the applied irrigation and actual climatic factors. The formulation for crop simulation is as follows:
First compute the final soil moisture with the following relation

$$\theta_i^k = \left(\theta_i^{k+1}ED_i^{k+1} + IRR_i^k - Fkc_i^k APET^k + ARF^k\right)/ ED_i^k \tag{14}$$

If $\theta_i^{k+1} < 3.1$

$$ED_i^{k+1}\theta_i^{k+1} = \frac{\left[\theta_i^k ED_i^k + IRR_i^{k+1} - \frac{Fkc_i^{k+1}APET^{k+1}}{2.0} + \frac{Fkc_i^{k+1}APET^{k+1}}{2.0}ZW + ARF^{k+1}\right]}{\left(ED_i^{k+1}\right)\frac{Fkc_i^{k+1}APET^{k+1}}{2.0}} \tag{15}$$

or

$$\theta_i^k = \theta_i^{k-1}\left[ED_i^{k-1} - \frac{Fkc_i^k APET}{2.0}\right] + \frac{Fkc_i^k APET}{2.0} Zw + ARF^k + IRR_i^k - \frac{Fkc_i^k APET}{2.0} \div ED_i^k \qquad (16)$$

or

$$\theta_i^k = \left\{\theta_i^{k-1}\left[ED_i^{k-1} - \frac{Fkc_i^k APET}{2.0}\right] + IRR_i^k + \frac{Fkc_i^k APET}{2.0} Zw\right\} \div \left(ED_i^k - \frac{Fkc_i^k APET}{2.0}\right) \qquad (17)$$

The computed soil moisture status of the crops is used in the next fortnight to compute the demand.

6. Stochastic Analysis of Evapotranspiration

It was previously stated that the data regarding the climatic factors is uncertain in nature and the determination of these factors beforehand is impossible. However, there is a general trend to assume the expected values for these factors and carry out the operation. The concept does not give a clear picture of the actual scenario and the appropriate weights for the individual growth stage of the crops are not assigned. The present study proposes a different method of forecasting the expected values for the climatic factors. The method of analysis starts with the computations of dependability values of reference evapotranspiration factors from the available data. The dependability of realisation of any stochastic variable is defined as the probability of equalling or exceeding that variable with a particular value.
Mathematically,

$$P(x \geq X) \qquad (18)$$

where P (.) is the probability and x is the variable under consideration and X is a stipulated value of the variable. A traditional method of estimation of the dependability value is the use of standard frequency formulae (e.g. Wiebull's formula or Hazen's formula). In the present study, a detailed probability analysis for the data is performed. The data is fitted to a standard probability distribution and the best fitting distribution is tested through the Kolmogorov Smirnov Test (Haan, 1977).

Once the values corresponding to different dependabilities are evaluated, dependability values for reference evapotranspiration are assumed to be different in different growth stages. The analysis is performed on the basis of the yield response factor. A high yield response factor signifies greater sensitivity towards the deficits, and thus, a higher level of dependability is assumed for the evapotranspiration data and a lower level of dependability is assumed for the rainfall data. This will ensure a higher value of irrigation required for the crop in the sensitive period. As a result, the crop will be safeguarded against any poor moisture content conditions.

7. LP Model Formulation for Optimal Cropping Pattern

At the start of each dry season, depending on the storage volume in the reservoir, the crop pattern must be determined. To evaluate the crop pattern, another LP model is used. In this model, irrigation depths are calculated from Eq. (6). The formulation is as follows:
The objective function is

$$MaxZ = C_1 X_1 + C_2 X_2 + C_3 X_3 \qquad (19)$$

which is subject to the following constraints:
1. Total available area

$$X_1 + X_2 + X_3 \leq A \qquad (20)$$

where X_1, X_2, and X_3 are the decision variables related to the area of individual crops; C_1, C_2, and C_3 are the cost coefficient for each crop in Indian Rupees (1 US $ = 50 INR); and
A is the maximum area available for irrigation.

2. Area of each individual crop:

The area under each crop is required to be constrained; thus, there are lower and upper bounds on the area under each crop. The lower bounds indicate the minimum area that can be allocated to a crop, while the upper bound indicates the maximum. In the present study, the lower bounds were defined for all the crops except cash crops, while the upper bounds were defined considering the present cropping pattern. The constraints can be expressed as

$$L_i \leq X_i \leq M_i \qquad (21)$$

where L_i corresponds to the lower bound of the area for the i^{th} crop and M_i corresponds to the upper bound on the area of the i^{th} crop.

8. Model Application

The developed models were applied to the Chiller reservoir system in Madhya Pradesh, India (Latitude 23°23' N and Longitude 76°18' E). In the central part of India, many reservoir projects have been constructed for irrigation, but no irrigation is available from these reservoirs during the monsoon period (from June to September). The area receives about 90 to 95 % of its rainfall during the Monsoon season. The rainfall then becomes runoff to the reservoirs. These reservoirs are designed to contain the runoff in the monsoon season, but there is no runoff during non-monsoon months. The present formulations are specially suited for these types of reservoirs. Non-monsoon rainfall is rare and provides little runoff. A systematic data base was prepared for the various physical features of the reservoirs, including the meteorological and hydrological data such as evapotransiration, details of crops in the command area, details of net returns from individual crops and soil properties collected from the College of Agriculture, Indore, India.

9. Results and Discussion

Optimum Crop Pattern

A separate computer program was run before the real time operation program to determine the optimum crop pattern for all possible storage values. The results of the optimum crop pattern are stated in Table 1. The results indicate that from a storage level of 31.10 M m^3 to a storage level of 26.06 M m^3, the cropping pattern is same as the one that has been adopted in the project formulation. However, below a storage level of 26.06 M m^3, the crop pattern changes suddenly, and wheat (ordinary) is not recommended by the model. The area of wheat (hybrid) also gets reduced when the rainfall storage is below this level. However, the area for Gram is full, up to a storage level of 15.83 M m^3. The change in cropping pattern indicates that efficient water usage is maintained.

Table 1 Optimum Cropping Pattern for Different Live Storage Values

Live storage (M m^3)	**Area (ha) for different crops**		
	Wheat (ordinary)	**Gram**	**Wheat (hybrid)**
4.3230	-	342.910	120.00
8.2379	-	427.580	500.00
12.3246	-	1084.015	500.00
15.8632	-	1100.000	855.00
20.7581	-	1100.000	1434.00
26.0986	300.0	1100.000	1700.00
28.8610	300.0	1100.000	1700.00
30.1250	300.0	1100.000	1700.00
31.1000	300.0	1100.000	1700.00

Results from Real-Time Operation Model

The real-time operation model gives an optimal operating policy for the available storage in the present fortnight considering the future. The model also yields the values of irrigation to be applied to individual crops in the fields. In the wake of deficient water supplies, the model distributes the available water over the time for different crops optimally. The sample results of the present model are stated in Table 2. The available moisture to the crops is not affected, and generally the soil remains at the upper limit of the available soil-moisture. This is because the crop pattern is predicted according to the availability of the storage in the reservoir. The results are indicative of successful application of the real-time operation strategy proposed in the present work.

Table 2 Sample Results Showing the Soil Moisture, Available Soil Moisture, Storage, and Irrigation to be applied for Different Crops for a Real-Time Reservoir Operation Model (LP)
Live Storage in the Reservoir 31.1 M m^3

PARAMETER	**FORTNIGHT**										
	1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**
Reservoir Storage (M m^3)	29.28	28.17	26.30	22.22	19.68	14.64	10.87	5.62	4.24	3.63	3.60
Crop	Wheat (ordinary)										
1) Soil Moisture (mm/cm)	3.76	3.89	3.84	3.07	3.54	3.30	3.22	3.17	4.0	- -.	- -
2) Available soil Moisture (mm/cm)	0.9	0.9	0.9	0.87	0.9	0.9	0.9	0.9	0.9	- -	- -
3) Applied Irrigation (mm)	53.62	90.63	92.87	36.04	163.9	8.44	23.02	19.94	102.6	- -	- -
Crop	GRAM										
1) Soil Moisture (mm/cm	3.90	3.07	3.28	3.15	3.4	3.28	3.66	3.23	3.47	- -	- -
2) Available soil Moisture (mm/cm)	0.9	0.87	0.9	0.9	0.9	0.9	0.9	0.9	0.9	- -	- -
3) Applied Irrigation (mm)	68.76	22.27	60.67	41.59	26.96	37.64	53.15	0.00	33.17	- -	- -
Crop	**Wheat (hybrid)**										
1) Soil Moisture (mm/cm	- -	- -	- -	4.00	3.06	3.48	3.32	3.28	3.38	3.18	3.19
2) Available soil Moisture (mm/cm)	- -	- -	- -	0.9	0.86	0.9	0.9	0.9	0.9	0.9	0.9
3) Applied Irrigation (mm)	- -	- -	- -	94.21	37.19	127.9	78.89	162.9	0.00	36.09	0.0

Relative Yield Ratios

Relative yield ratios computed for different crops at different live storage values are shown in Table 3. The relative yield ratios for all the crops become one if live storage in the reservoir is equal to or greater than 28.89 M m^3. The GA model is found to be better for application in real world operation of the reservoir.

Table 3 Relative Yield Ratio for Different Live Storage Values Computed With a Real-Time Reservoir Operation Model

Live storage (M m^3)	**Relative yield ratio for different crops LP**		
	Wheat (ordinary)	**Gram**	**Wheat (hybrid)**
4.3230	-	0.9677	1.000
8.2362	-	0.9083	1.000
12.3246	-	0.9576	1.000
15.8632	-	0.989	1.000
20.7581	-	0.987	0.911
26.0986	1.000	0.987	0.952
28.8610	1.000	0.987	1.000
30.1250	1.000	1.000	1.000
31.1000	1.000	1.000	1.000

10. Conclusion

A real-time model using an integrated Linear Programming Model for a reservoir system meant for irrigation has been developed in the present study to obtain an optimal reservoir operating policy that incorporates field level decisions, while also deciding the appropriate time and amount of water to release from the reservoir.

From the analysis, the following conclusions can be drawn:

The developed model can be successfully applied to irrigation supporting reservoir systems. Furthermore, the models ensure an optimum reservoir release over different time periods. In addition, they also ensure optimum allocation of the available water over the different crops in the fields. While allocating the water to different crops in the fields, the model takes into account the critical growth stages of the crops and allocates sufficient water to each crop to safeguard it against any ill effects of water deficits. The optimum crop pattern model used in the study will only allow productive irrigation, so the amount of wasted water is reduced.

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Nomenclature

AET_i^k	Actual evapotranspiration in period *k* from crop *i* (mm)
$APET^k$	Actually occurring potential evapotranspiration in period *k* (mm)
ARF^k	Actual rainfall value in the fortnight *k*
A^k and B^K	Constants relating the storage to reservoir evaporation
A_o	Area of spread at dead storage level
d	Depletion factor
ED_i^k	Effective root zone depth of a crop *i* in period *k* (cm)
ED_i^{k+1}	Effective root zone depth of a crop *i* in period *k+1* (cm)
Eff	Overall efficiency
Fkc_i^k	Crop evapotranspiration coefficient
ID	Industrial supply from the reservoir (mandatory release)
IRR_i^k	Irrigation applied to crop *i* in stage *k* (mm)
Ky^k	Yield response factors for a crop *i* in period *k*
PET_i^k	Potential evapotranspiration in a particular geographical location (mm)
RE^k	Rate of evaporation in fortnight *k*
RF^k	Rainfall in period *k* (mm)
S^k	Reservoir storage at the beginning of period *k*
S^{k+1}	Reservoir storage at the end of period *k*
Zf	Field capacity for the soil (mm/cm)
Zw	Permanent wilting point for the soil (mm/cm)
Zww	Critical available moisture limit (mm/cm)
θ_i^k	Initial soil moisture in the time stage *k* in for a crop *i* (mm/cm)
θ_i^{k+1}	Final soil moisture in a particular time stage *k* for a particular crop *i* (mm/cm)
Y_{ai}	Actual crop yield
Y_{mi}	Maximum crop yield

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