

## EFFECT OF DEPOSITION THICKNESS ON THE INCIPIENT MOTION OF SEDIMENTS

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### ABSTRACT

The current literature on incipient motion are largely for loose boundary channel with unlimited depth of sediment as compares to rigid boundary channel with limited sediment depth. A few existing literature for rigid boundary channel suggested that the incipient motion value is substantially lower than for loose boundary channels for any particle size. Some existing literature also shown that the current available equations for incipient motion in rigid boundary channel became less accurate as the sediment deposition thickness increased. Nevertheless, the understanding on how the deposition thickness could affect the incipient motion of sediment is still lacking in the literature and need further study. The current study highlights the effect of sediment deposition thickness on incipient motion by conducting experimental work in a rigid rectangular flume of 0.6 m wide. Results revealed that sediment deposits thickness have effect on the incipient motion of the sediment particle at low sediment deposits thickness and the effect will diminished with the increased in thickness of the sediment deposits. It was also observed from the current study that the sediment deposits start to behave like a loose boundary for thickness above 48 mm. A new equation was proposed in the current study to predict critical velocity during incipient motion by incorporating the sediment deposits thickness. This new equation appears to be consistent and was not affected by the sediment deposits thickness and is an attempt toward unifying the equations for both rigid boundary and loose boundary conditions.

*Keywords:* Critical velocity, Deposition thickness, Incipient motion, Sediment

### 1. INTRODUCTION

The initial motion of sediment particles is commonly called incipient motion while the condition that is just adequate to initiate sediment motion is termed critical condition (Dey & Papanicolaou, 2008). For the purpose of design, the Shields diagram (Shields, 1936) was widely used to predict incipient motion of granular particles especially for loose boundary channel such as alluvial channel (Vongvisessomjai et al., 2010). For rigid boundary channel, there is a limitation in terms of depth of sediment and source of new sediment for transport (Butler et al., 1996). Despite the very different boundary conditions in sewers (Ashley et al., 2004) which are of rigid boundary, the Shields diagram has been applied in a number of studies on sewer and storm sewer (Laplace et al., 1992; Verbanck et al., 1994; Almedeij, 2012). The Shields diagram was developed using a relationship (see Eq. [1]) based on the balance between particle weight and boundary shear stress:

$$\theta_c = \frac{\tau_c}{gd(\rho_s - \rho)} = f\left(\frac{u_* d}{\nu}\right) = f(\text{Re}_*) \quad [1]$$

where  $\theta_c$  is the dimensionless Shields stress;  $\tau_c$  is the critical shear stress [N/m<sup>2</sup>];  $g$  is the gravitational acceleration [m/s<sup>2</sup>];  $\rho_s$  is the sediment density [kg/m<sup>3</sup>];  $\rho$  is the fluid density [kg/m<sup>3</sup>];  $d$  is the grain size (normally  $d = d_{50}$  for uniform sediment) [m];  $u_* = \sqrt{\tau_c / \rho}$  is the shear velocity [m/s];  $\nu$  is the kinematic viscosity of fluid [m<sup>2</sup>/s] and  $\text{Re}_*$  is the dimensionless grain Reynolds number. Rewriting Eq. [1] in terms of particle critical Froude number  $F_d$  by assuming the Shields' criterion  $\tau_c / gd(\rho_s - \rho) = F_d^2 = 0.056$  and Manning's  $n = 0.04d^{1/6}$  ( $d$  in meters) and for wide channel, Eq. [2] was obtained (Novak & Nalluri, 1975):

$$F_d = \frac{V_c}{\sqrt{gd(S_s - 1)}} = 1.92 \left(\frac{d}{y_0}\right)^{-0.167} \quad [2]$$

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in which  $V_c$  is the critical velocity [m/s];  $g$  is the gravitational acceleration [m/s<sup>2</sup>];  $d$  is the grain size (normally  $d = d_{50}$  for uniform sediment) [m];  $S_s$  is the specific gravity of the sediment and  $y_0$  is the normal flow depth of flow in channel.

Existing literatures (Novak & Nalluri, 1975; Bong et al., 2013) on rigid smooth bed channels have shown that the critical velocity and critical shear stress for incipient motion are substantially lower for any particle size than for loose boundary channels. Existing equations in the literature developed to predict critical velocity for rigid boundary channel are usually in the form of Eq. [3] (Novak & Nalluri, 1984; El-Zaemey, 1991):

$$\frac{V_c}{\sqrt{gd(S_s - 1)}} = a \left( \frac{d}{R} \right)^b \quad [3]$$

where  $V_c$  is the critical velocity [m/s];  $g$  is the gravitational acceleration [m/s<sup>2</sup>];  $d$  is the grain size (normally  $d = d_{50}$  for uniform sediment) [m];  $S_s$  is the specific gravity of the sediment;  $R$  is the hydraulic radius of the flow section;  $a$  and  $b$  are coefficients. Novak and Nalluri (1984) defined the  $a$  and  $b$  as 0.5 and -0.4; while El-Zaemey (1991) as 0.75 and -0.34 respectively. Some literatures suggested that Eq. [3] which was developed for rigid boundary channel with limited sediment depth became less accurate as the sediment deposit thickness increased (Ab. Ghani et al., 1999; Bong, et al., 2013; Salem, 2013). It was also observed that as the thickness of sediment deposits increased, the critical velocity required for incipient motion also increased (Bong, et al., 2013). This could be due to the effect of 'support' from neighboring particles increases as the sediment deposits thickness increased; resulting in greater friction between sediment particles and higher critical velocity to move the particles (Bong, et al., 2013).

Hence, it is apparent that the use of Shields' threshold criterion for self-cleansing design of sewer and storm sewer with limited sediment deposits thickness will produce significant errors. Sediment deposits in combined sewers generally has limited thickness from less than 10 mm to 60 mm (Lange & Wichern, 2013) and up to 100 mm (Ashley et al., 1992). As for rectangular open storm sewer, the sediment deposits thickness could range from 10 mm to 330 mm (Bong et al., 2014). Conversely, the existing critical velocity equations for rigid boundary channels did not incorporate the effect of sediment deposits thickness, rendering it inaccurate with the increasing of sediment deposits thickness.

This paper aims to understand the effect of sediment deposits thickness on incipient motion which is still lacking in the literature. Data were obtained through incipient motion experiment in a rigid rectangular flume by varying the sediment deposits thickness. New critical velocity equation was proposed by incorporating the effect of sediment deposits thickness.

## 2. DIMENSIONAL ANALYSIS

A two-phase phenomenon involving fluid and sediment such as the study of incipient motion can be described by three components, namely i) fluid; ii) non-cohesive granular medium; and iii) flow (Yalin, 1977). The fluid is defined by its density  $\rho$  [kg/m<sup>3</sup>]; the non-cohesive granular medium is defined by its density  $\rho_s$  [kg/m<sup>3</sup>] and size  $d$  (normally  $d = d_{50}$  for uniform sediment) [m]; and the flow is defined by the hydraulic radius of flow area  $R$  [m] and gravity acceleration  $g$  [m<sup>2</sup>/s] (Bong, et al., 2013). To incorporate sediment deposition thickness, the sediment deposit thickness  $t_s$  [m] and the normal flow depth  $y_0$  [m] (see Figure 1) can be included in the analysis.

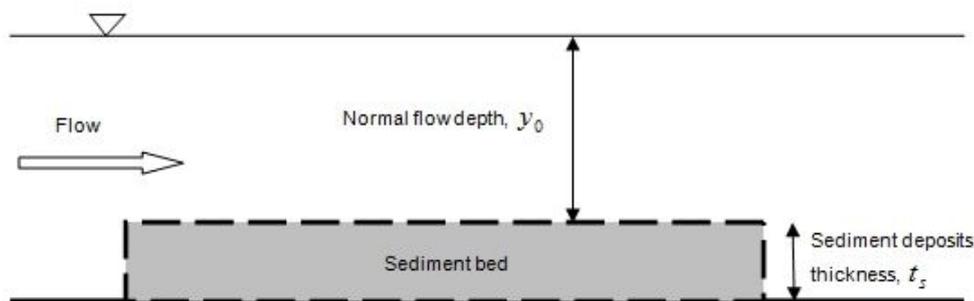


Figure 1. Depths definition for the current study.

Choosing to replace  $g$  with  $\gamma_s = g(\rho_s - \rho)$  where the specific weight for the sediment  $\gamma_s$  can be excluded as it would be a constant and can be eliminated from the analysis (Azmathullah et al., 2005), the dimensionless terms of the incipient motion function are given by:

$$\frac{V_c}{\sqrt{gd(S_s - 1)}} = f\left(\frac{d}{R}, \frac{t_s}{d}, \frac{t_s}{y_0}\right) \quad [4]$$

### 3. METHODOLOGY

The current study is an extension of previous work by Bong et al. (2013) and involved a rectangular flume as shown in Figure 2. The incipient motion experiment was conducted with six sediment deposits thickness namely one layer ( $t_s = d_{50}$ ), 5 mm, 10 mm, 24 mm, 48 mm and 100 mm. The experiment was initially conducted with a flume slope of 0.001 and was repeated for a flume slope of 0.002 to reconfirm the results for different slope values. The non-cohesive uniform sediment used had median  $d_{50}$  sizes of 0.81 mm, 1.53 mm and 4.78 mm with specific gravity of 2.54, 2.55 and 2.57 respectively. The definition for incipient motion used in the current study was of general movement (Kramer, 1935). During the experiment, water level and discharge were slightly increased by controlling the pump that supplies water into the flume until incipient motion was observed. The velocity and discharge values during incipient motion were obtained from an electronic flow meter. The range of experimental parameters for the current study is as shown in Table 1.

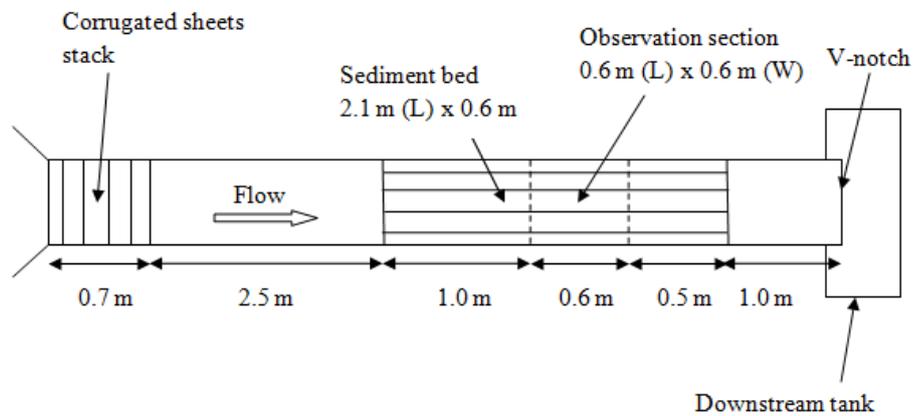


Figure 2. Schematic diagram of the flume used in the current study (not to scale).

Table 1. Range of experimental parameters for current study.

Parameter	Current study
Flume width $W$ (m)	0.6
Critical velocity $V_c$ (m/s)	0.216 – 0.632
Normal flow depth $y_0$ (m)	0.006 – 0.132
Reynolds number $Re$	7878.90 – 163512.50
Flume slope $S_0$	0.001 and 0.002
Sediment median size $d_{50}$ (mm)	0.81 – 4.78
Sediment specific gravity $S_s$	2.54 – 2.57
Sediment deposit thickness $t_s$ (mm)	0.81 - 100

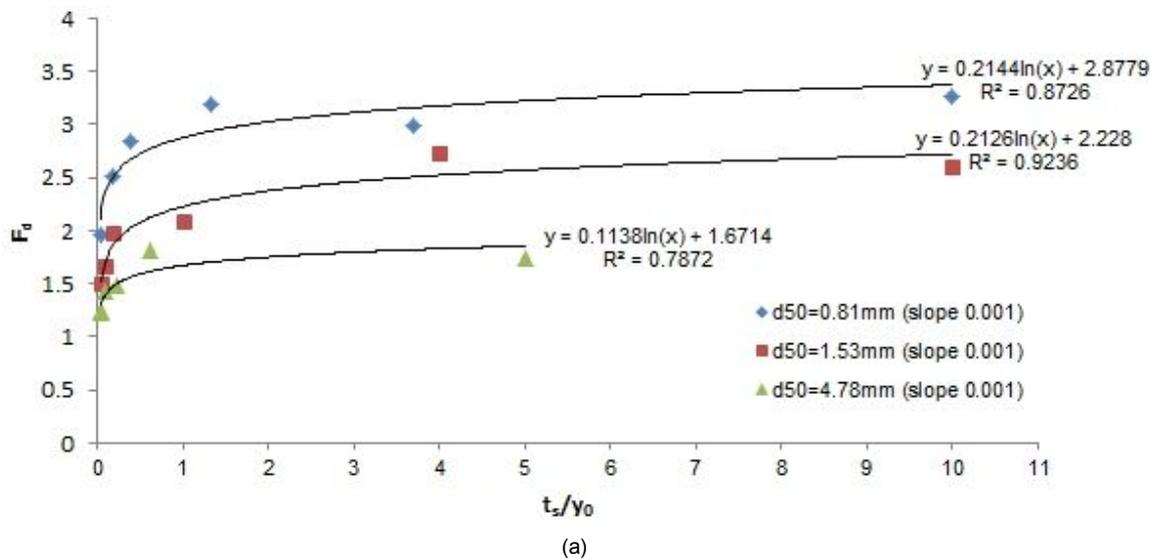
Using the data from the current study, improvement to the existing critical velocity equation was done by performing multiple linear regressions on the dimensionless terms in Eq. [4]. A total number of 36 data obtained from the current study was used for the multiple linear regressions analysis. The best regression model from the combination of dimensionless groups in Eq. [4] was selected based on four criteria: (a) coefficient of determination  $R^2$ ; (b) adjusted  $R_{adj}^2$ ; (c) mean square error  $MSE$ ; and (d) Mallow's  $C_p$  statistics. Performance test on the new equation was done by calculating the discrepancy ratio using Eq. [5] with the acceptable range of 0.5 to 2.0 which is normally used for the study on sediment transport (Yang, 1996).

$$\text{Discrepancy ratio} = \frac{V_c \text{ predicted (m/s)}}{V_c \text{ observed (m/s)}} \quad [5]$$

#### 4. RESULTS AND DISCUSSION

##### 4.1 Effects of sediment deposits thickness

Figure 3 shows the effect of sediment deposits thickness on incipient motion by plotting the  $F_d$  against  $t_s/y_0$ . It was observed that  $F_d$  increases but at diminishing rate as  $t_s/y_0$  increases for both the flume slopes used in the current study. This shows that sediment deposits thickness have effect on the incipient motion of the sediment particle at low sediment deposits thickness and the effect will diminished with the increased in thickness of the sediment deposits. The equations in both the graphs in Figure 3 showed that the relationship between  $F_d$  and  $t_s/y_0$  are best fitted ( $R^2$  value close to unity) with logarithmic relationships. This effect could be due to increment of friction that existed between the sediment particles with the increment in thickness. At thicker sediment deposits, the increment of friction between the sediment particles is negligible with further increase in deposits thickness. Consequently, this effect causes the existing equations in the literature for incipient motion in rigid boundary channel (Novak & Nalluri, 1984; El-Zaemey, 1991) which are for limited sediment deposits thickness to be less accurate as the sediment deposits thickness increased since the equations did not take into account the sediment bed thickness.



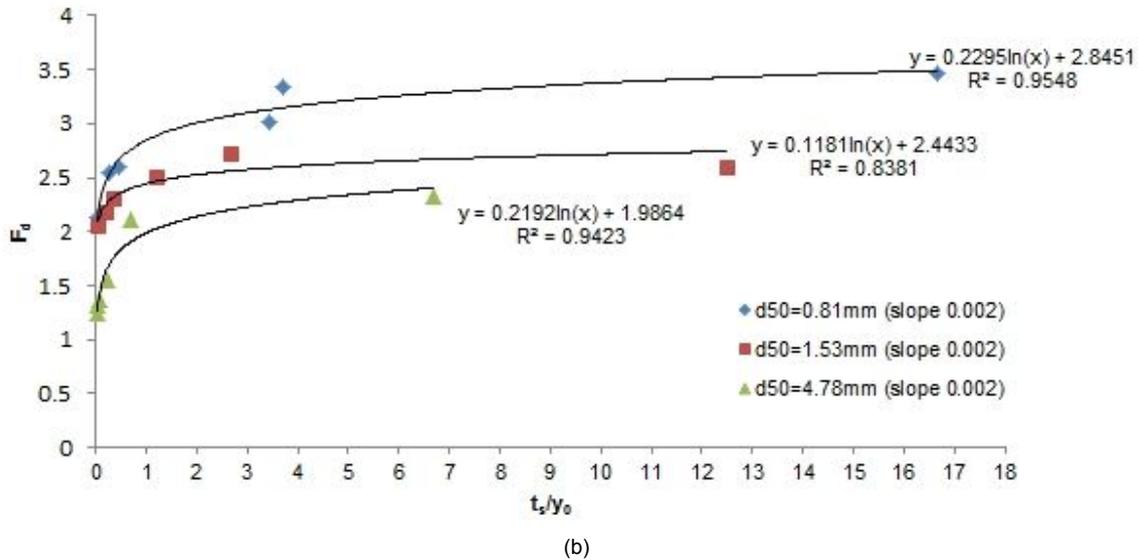


Figure 3.  $F_d$  versus  $t_s/y_0$  for: a) flume slope of 0.001; and b) flume slope of 0.002.

Using Eq. [2] which expressed the Shields' criterion in terms of  $F_d$ , the graph in Figure 4 which shows the relationship between  $F_d$  with  $d/y_0$  was obtained. The  $F_d$  from Shields' criterion was plotted as a line in Figure 4. Comparing the  $F_d$  calculated from the data of the current study with  $F_d$  from Shields' criterion, it was observed that as the sediment deposits thickness increases,  $F_d$  from the current study became closer to the  $F_d$  for Shields' criterion. It was also observed that most of the points for  $F_d$  from the current study for sediment deposits thickness of 48 mm and 100 mm either touches or above the Shields' criterion line. This trend was observed for both the flume slopes used in the current study. This showed that the sediment deposits start to behave like a loose boundary for thickness above 48 mm.

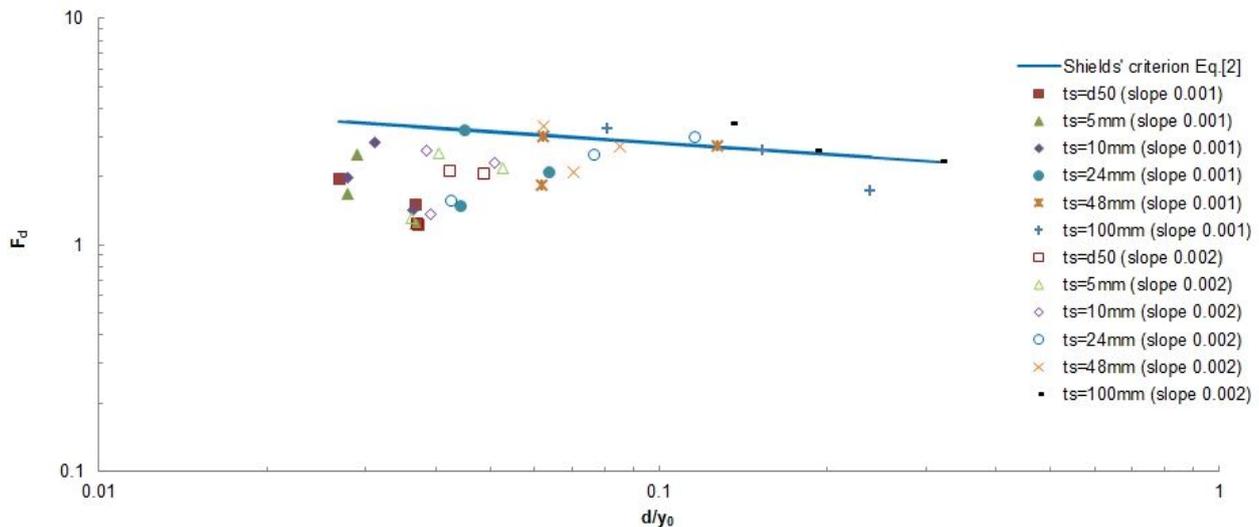


Figure 4.  $F_d$  versus  $d/y_0$  for flume slopes of 0.001 and 0.002.

#### 4.2 Improvement to critical velocity equation

From the previous Section 4.1, it was obvious that sediment deposits thickness has effect on incipient motion for rigid boundary channel. This effect can be included into existing critical velocity equations for rigid boundary channel (Novak & Nalluri, 1984; El-Zaemey, 1991) by incorporating the dimensionless terms  $t_s/d$  and  $t_s/y_0$  as mentioned in the previous Section 2 and Eq. [4]. Table 2 shows the results of Pearson correlation analysis between  $F_d$  with  $t_s/d$  and  $t_s/y_0$ . Results of the Pearson correlation analysis showed that both  $t_s/d$  and  $t_s/y_0$  have strong correlation (having correlation value of more than 0.7) with  $F_d$  and significant ( $p$ -value = 0.000).

Table 2. Correlation analysis for  $F_d$  with  $t_s/d$  and  $t_s/y_0$ .

Dimensionless term	$F_d$
$t_s/d$	0.813 ( $p$ -value = 0.000)
$t_s/y_0$	0.749 ( $p$ -value = 0.000)

Multiple linear regressions were performed and the results were as shown in Table 3. From Table 3, Eq. [9] was the best among the regression models that incorporate the dimensionless terms  $t_s/d$  and  $t_s/y_0$  in the current study. Eq. [9] has  $R^2$  and  $R_{adj}^2$  values closest to unity among the regression models in the current study and having the smallest  $MSE$  value together with  $C_p$  value that is the same with the number of terms in the model.

Table 3. Results of multiple linear regressions incorporating  $t_s/d$  and  $t_s/y_0$ .

Equation	$R^2$	$R_{adj}^2$	$MSE$	$C_p$	Eq.
$\frac{V_c}{\sqrt{gd(S_s - 1)}} = 0.85 \left(\frac{d}{R}\right)^{-0.18} \left(\frac{t_s}{d}\right)^{0.21}$	0.733	0.717	0.00518	77.3	[6]
$\frac{V_c}{\sqrt{gd(S_s - 1)}} = 0.87 \left(\frac{d}{R}\right)^{-0.38} \left(\frac{t_s}{y_0}\right)^{0.20}$	0.779	0.765	0.00429	58.9	[7]
$\frac{V_c}{\sqrt{gd(S_s - 1)}} = 0.99 \left(\frac{t_s}{d}\right)^{0.32} \left(\frac{t_s}{y_0}\right)^{-0.12}$	0.694	0.675	0.00594	3.0	[8]
$\frac{V_c}{\sqrt{gd(S_s - 1)}} = 1.38 \left(\frac{d}{R}\right)^{-1.65} \left(\frac{t_s}{d}\right)^{-1.39} \left(\frac{t_s}{y_0}\right)^{1.51}$	0.920	0.913	0.00159	4.0	[9]

Eq. [9] was selected for further performance test by calculating the discrepancy ratio between the  $V_c$  predicted by Eq. [9] with the  $V_c$  observed from the experiment in the current study using Eq. [5]. The number of values within the acceptable range of 0.5 to 2.0 was noted. As comparison, performance test were also done using equations by Novak and Nalluri (1984) and El-Zaemey (1991). Figure 5 (a), Figure 5(b) and Figure 5 (c) show the comparison between observed and predicted critical velocity using data from the current study for equations by Novak and Nalluri (1984), El-Zaemey (1991) and Eq. [9] respectively. Results from Figure 5 for the performance test are as summarized in Table 4. From Table 4, it was observed that Eq. [9] performed better than the equations by Novak and Nalluri (1984) and El-Zaemey (1991) by having all the predicted values within the acceptable range.

Table 4. Comparison of results from performance test.

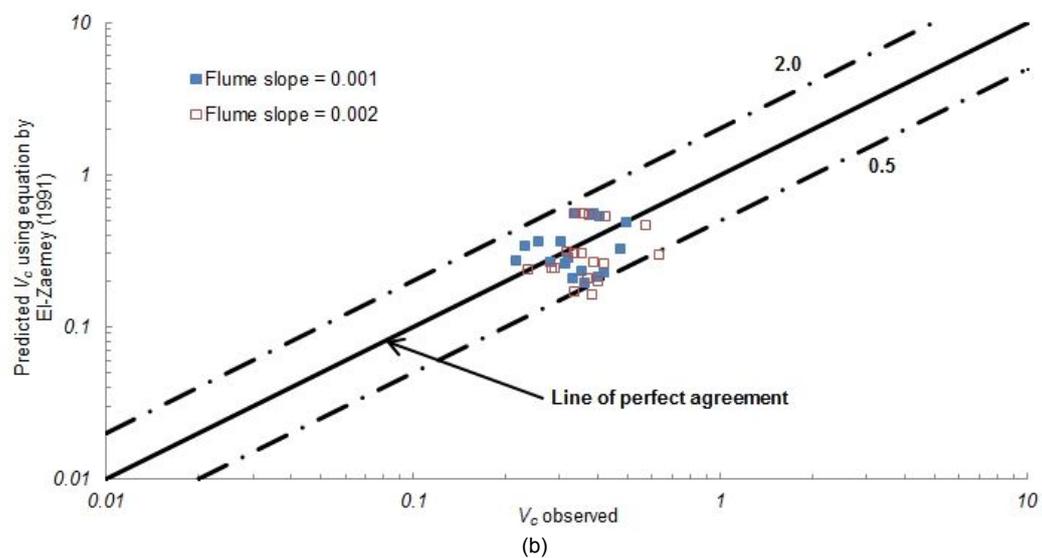
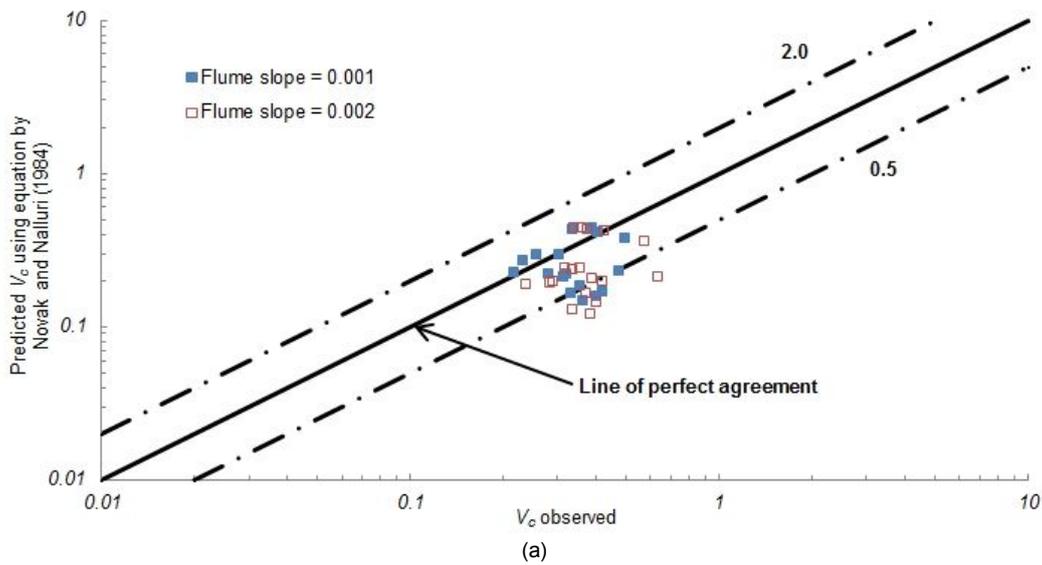
Equation	Values within acceptable discrepancy ratio (0.5 – 2.0)	
	Number of data	Percentage of data (%)
Novak and Nalluri (1984)	25	69.4
El-Zaemey (1991)	34	94.4
Eq. [9]	36	100

Performance test analysis of Eq. [9] as well as equations by Novak and Nalluri (1984) and El-Zaemey (1991) in terms of various sediment deposits thickness are as shown in Table 5. From Table 5, it was observed that as the sediment

deposits became thicker, equations by Novak and Nalluri (1984) and El-Zaemey (1991) became less accurate with the discrepancy ratio values became further from unity. Eq. [9] from the current study appears to be consistent and was not affected by the sediment deposits thickness. Hence, the new Eq. [9] proposed in the current study brought improvement in terms of the prediction power for critical velocity values as compared to the existing equations in the literature.

Table 5. Comparison of results from performance test.

Equation	Discrepancy ratio for various sediment deposits thickness $t_s$						Average
	$d_{50}$	5 mm	10 mm	24 mm	48 mm	100 mm	
Novak and Nalluri (1984)	1.073	0.984	0.891	0.698	0.539	0.389	0.762
El-Zaemey (1991)	1.337	1.225	1.109	0.890	0.696	0.526	0.964
Eq. [9]	0.986	0.991	0.986	1.003	0.968	1.040	0.996



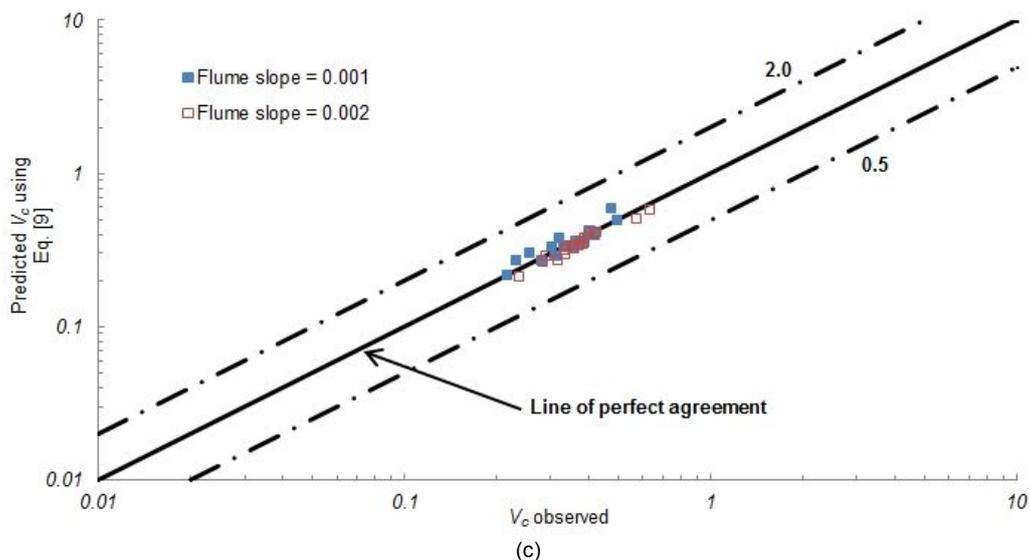


Figure 5. Comparison between observed and predicted critical velocity for equations by (a) Novak and Nalluri (1984); (b) El-Zaemey (1991); and (c) Eq. [9].

## 5. CONCLUSIONS

The current study aims to understand the effect of sediment deposits thickness on incipient motion. Experimental work on incipient motion was conducted in a rectangular flume by varying the thickness of sediment deposits. Results show that sediment deposits thickness have effect on the incipient motion of the sediment particle at low sediment deposits thickness and the effect will diminished with the increased in thickness of the sediment deposits. Also observed from the current study was that thicker sediment deposits of 48 mm and 100 mm behaved according to the Shields' criterion for loose boundary channel. Existing incipient motion equation for rigid boundary channel became less accurate as the sediment deposits thickness increases. A new equation was proposed in the current study to predict critical velocity during incipient motion by incorporating the sediment deposits thickness. This new equation appears to be consistent and was not affected by the sediment deposits thickness. The new equation brought improvement in terms of the prediction power for critical velocity values as compared to the existing equations in the literature and is an attempt toward unifying the equations for both rigid boundary and loose boundary conditions.

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