

NUMERICAL SIMULATION OF MUDFLOW ON STEEP SLOPE TRIGGERED BY HEAVY RAINFALL

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Abstract

A preliminary study on mudflow triggered by heavy rainfall on steep slope is carried out in this study. A one-dimensional depth averaged model is developed to simulate the dam-break flow of mudflow on a dry, inclined channel. The constitutive law based on Herschel-Bulkley model is used to simulate the rheological behavior of mudflow. Harten's Total Variation Diminishing (TVD) scheme is adopted in the numerical model to improve shock wave capturing and suppress oscillations. The numerical model is validated against experiment result from literature study by comparing the propagation of shock front and flow profile. It is shown that the numerical model could reproduce the characteristic of mud flow satisfactorily. The development of a one-dimensional depth averaged model is vital as it will become the basis for the development of a two-dimensional depth averaged model, which is capable of estimating the extent of mudflow hazard.

Key Words: mudflow, yield stress, Herschel-Bulkley, numerical simulation

1. INTRODUCTION

During heavy rainfall, considerable amount of silt, organic matter, sand and granular material are picked up by runoff on bare slopes. The flow can be classified as water floods, mud floods, mudflow or landslide depending on the concentration of sediment in the fluid. For hyper-concentrated fluid, the flow is termed mud-flow if the content of clay particle ($\leq 40\mu\text{m}$) is high, and the term debris flow is used when the content is low. (Laigle and Coussot, 1988).

The numerical simulation of mudflow focuses on the selection of suitable constitutive relation. The Bingham model is widely used to represent the constitutive relation for mudflow and its application can be seen in the work of Jeyapalan et. al. (1983) in simulating dam break flow problem of mine tailing material. Schamber et. al (1995) also adopted Bingham model in their studies of mudflow routing by using a one-dimensional model solved with the method of characteristics. Bingham model is found more suitable to model the behavior of mudflow under low shear rate. For high shear rate of clay-water suspension, the Herschel-Bulkley model could produce the behavior of the flow well (Govier and Aziz, 1982). Coussot and Piau, (1994) showed that the Herschel-Bulkley model describes the behavior of fine mud suspension well. The adoption of Herschel-Bulkley constitutive model can be seen in the numerical work of Laigle and Coussot (1997), Rickermann et. al. (2006) and Schippa and Pavan (2011).

In this preliminary study, a one-dimensional numerical model adopting higher order numerical scheme is used to simulate the release of hyper-concentrated flow of fine suspension as representation of mudflow at mountainous area which is made up of a large amount of fine particle below $100\mu\text{m}$ (Coussot and Piau, 1994). The friction slope formulated empirically for uniform laminar flow by Coussot and Piau (1994) based on the Herschel-Bulkley model is used. Initially, the simulation of dam-break flow on dry bed is carried out to test the numerical model accuracy and stability of the model. Subsequently, the simulation of the release of fine mud suspension held in a reservoir on an inclined dry channel is carried out. The numerical result is verified against the experiment carried out by Laigle and Coussot (1997).

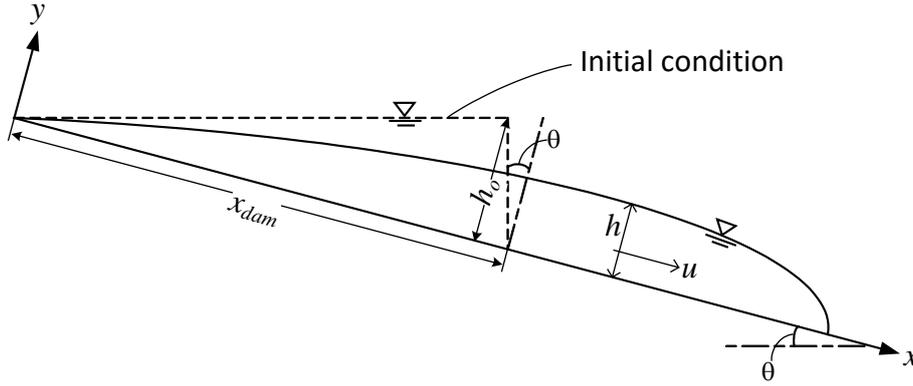


Fig 1. The release of finite volume of fine suspension onto an inclined dry bed

2. NUMERICAL SIMULATION

2.1 Governing equations

The one dimensional shallow water equations which are comprised of the continuity and momentum equations are used to simulate the release of a finite volume of fine suspension on an inclined dry bed as shown in Figure 1. Based on Figure 1, the one dimensional shallow water equations can be written as follows,

Continuity equation

$$\frac{\partial h}{\partial t} + \frac{\partial q}{\partial x} = 0 \quad (1)$$

Momentum equation

$$\frac{\partial q}{\partial t} + \frac{\partial uq}{\partial x} + \frac{1}{2} g \cos \theta \frac{\partial h^2}{\partial x} = gh \left(\sin \theta - \frac{\tau_{bx}}{\rho g R} \right) \quad (2)$$

Here, q is the discharge per unit width, h is the flow depth, θ is the bottom slope of the bed, R is the hydraulic radius, τ_{bx} is the bottom or wall shear stress in x direction, ρ is the fluid density, g is gravitational acceleration and β is the momentum coefficient and is assumed as unity for simplicity.

2.2 Constitutive relation

For the simulation of hyper-concentrated fine suspension fluid, Coussot (1994) showed that the flow characteristics can be reproduced by using Herschel-Bulkley model satisfactorily. For the case of laminar flow of Herschel-Bulkley fluid in a rectangular channel, the bottom and wall shear stress can be expressed as follows (Coussot, 1994):

$$\tau_{bx} = \tau_y \left[1 + a (H_b)^{-0.9} \right] \quad (3)$$

where $a = 1.93 - 0.43 \tan^{-1} \left[\left(\frac{10h}{B} \right)^{20} \right]$, $H_b = \frac{\tau_y}{K} \left(\frac{h}{u} \right)^{1/3}$

Here B is the width of the rectangular channel, τ_y is the yield stress and K is the Herschel-Bulkley fluid parameter.

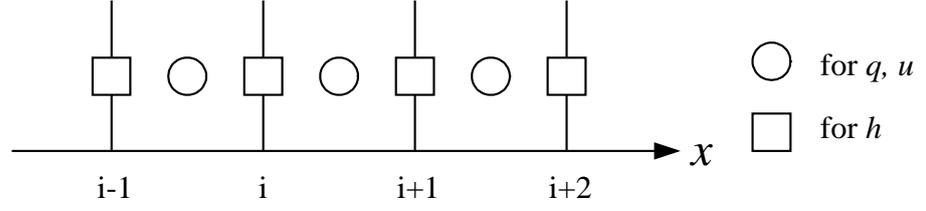


Fig 2. Definition of variables in the mesh system

2.3 Numerical scheme

The governing equations of Eq. (1) and Eq. (2) are solved numerically by using the finite volume method. The staggered mesh system is adopted where the variables are defined such as in Figure 2. Based on this mesh system, the governing equations are discretized as follows:

Continuity equation

$$\frac{h_{i+1/2}^{n+1} - h_{i+1/2}^n}{\Delta t} + \frac{q_{i+1}^n - q_i^n}{\Delta x} = 0 \quad (4)$$

Momentum equation

$$\frac{q_i^{n+1} - q_i^n}{\Delta t} + \frac{f_{i+1/2}^n - f_{i-1/2}^n}{\Delta x} + g \frac{(h_{i+1/2}^n)^2 - (h_{i-1/2}^n)^2}{2\Delta x} = gh_{ave} \left(\sin \theta - \frac{\text{sign}(u_i) \tau_{bx_i}}{\rho g R_{ave}} \right) \quad (5)$$

$$\text{sign}(u_i) = \begin{cases} 1 & \text{if } u_i > 1 \\ -1 & \text{if } u_i < 1 \end{cases}, h_{ave} = \frac{1}{2}(h_{i+1/2} + h_{i-1/2}), R_{ave} = \frac{1}{2}(R_{i+1/2} + R_{i-1/2}) \quad (6)$$

$$\tau_{bx_i} = \tau_y \left[1 + a (H_b)^{-0.9} \right], a = 1.93 - 0.43 \tan^{-1} \left[\left(\frac{10h_{ave}}{B} \right)^{20} \right], H_b = \frac{\tau_y}{K} \left(\frac{h_{ave}}{u_i} \right)^{1/3}$$

The term $f_{i+1/2}^n = (uq)_{i+1/2}$ is the flux across the control volume boundary at $i + 1/2$. A higher order TVD scheme is used to calculate this flux, by using MUSCL differencing method with minmod limiter (Tannehill et. al. 1997). Upon solving Eq. (4) and Eq. (5), the new value for the velocity is calculated as follows,

$$u_i^{n+1} = \frac{q_i^{n+1}}{h_{i-1/2+c}^{n+1}}, c = \begin{cases} 0 & \text{for } q_i^n \geq 0 \\ 1 & \text{for } q_i^n < 0 \end{cases} \quad (7)$$

2.4 Verification of numerical model

The simulation of an idealized dam break flow problem over a dry bed is carried out to check the numerical simulation stability and accuracy. A finite volume fluid with initial depth of $h_o = 1.5m$ and width $L_o = 5.0m$ is released instantaneously onto a dry horizontal bed. The mesh size is set as $\Delta x = 0.1m$ and time interval as $\Delta t = 0.0001s$. The CFL ratio is checked so that it is less than 0.75 for numerical stability. The performance of the numerical model is compared with the Ritter's solution in Figure 3. Before the first receding wave reaches the upstream wall at $x = 0m$, the flow profile between the receding wave and the front wave is given as follows (Jain, 2001):

$$h(x, t) = \frac{1}{9g} \left(2\sqrt{gh_o} - \frac{x - L_o}{t} \right)^2 \quad (8)$$

The numerical model could produce flow profile satisfactorily. However, numerical oscillation can be seen at the front wave due to discontinuity of flow.

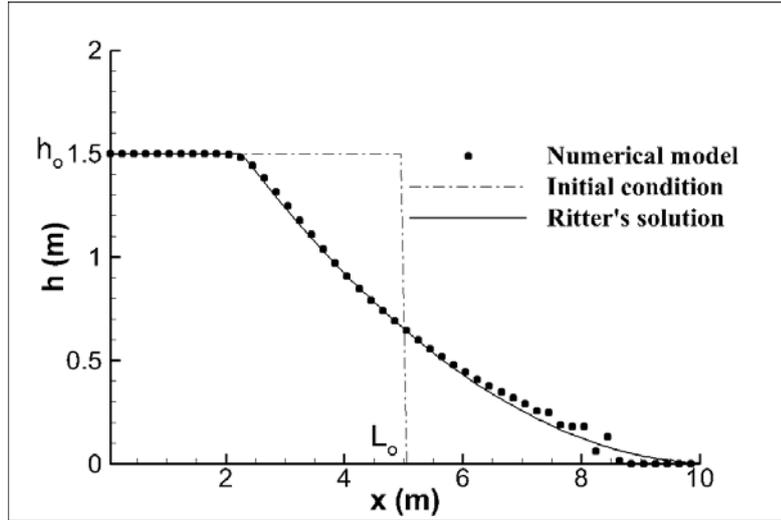


Fig 3. Numerical and analytical solution for dam break flow over dry bed at $t=0.7s$.

	Numerical model (A)	Experiment (C)	Error (%) (A-C)/A
Gauge 1 (1.65m)			
t_a (s)	0.54	0.48	11.1
y_p (cm)	2.33	2.56	-9.9
y_d (cm)	0.97	0.69	28.9
Gauge 2 (2.75m)			
t_a (s)	1.27	1.38	-8.7
y_p (cm)	2.09	2.21	-5.7
y_d (cm)	0.97	0.94	3.1
Gauge 3 (3.85m)			
t_a (s)	2.00	2.13	-6.5
y_p (cm)	1.81	1.87	-3.0
y_d (cm)	0.99	1.06	-7.0

Table - 1 Comparison of arrival time t_a , peak depth y_p and residual depth y_d at three gauges.

2.5 Simulation of mud flow on an inclined bed.

The numerical model is used to simulate the release of mud flow on an inclined bed based on the experiment setup by Laigle and Coussot (1997). In the experiment, a finite volume of water-clay mixture retained behind a dam gate is released instantaneously onto an inclined rectangular channel. The width of the rectangular channel is 0.6m and the slope of the channel is set at 16%. The water-clay mixture is contained between the dam-gate (at abscissa $x = 0.85m = x_{dam}$) and the upstream channel wall (at abscissa $x = 0.00m$). The maximum depth of water-clay mixture in the reservoir is $h_o = 0.13m$. The initial setup is shown in Figure 1. For the simulation of mud flow, the mesh size is set as $\Delta x = 0.01m$ and time interval as $\Delta t = 1.0 \times 10^{-6} s$. The water-clay mixture which follows the Herschel-Bulkley model has the following rheological parameters: yield stress $\tau_y = 19.0 Pa$ and consistency $K = 3.5 Pa \cdot s^{1/3}$. The density of the mixture is $\rho = 1410 kgm^{-3}$. The flow depth gauges are installed at abscissa $x = 1.65m$, $2.75m$ and $3.85m$.

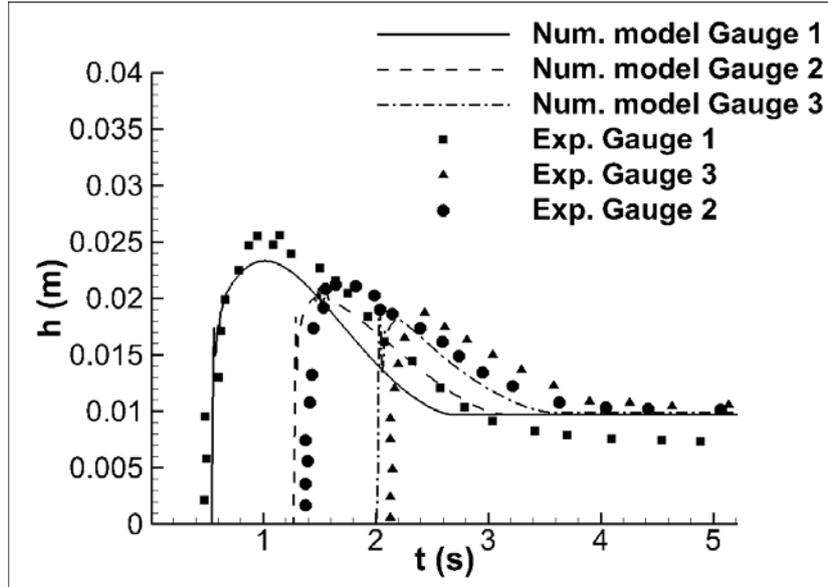


Fig 4. Time variation of flow depth at Gauge 1, 2 and 3.

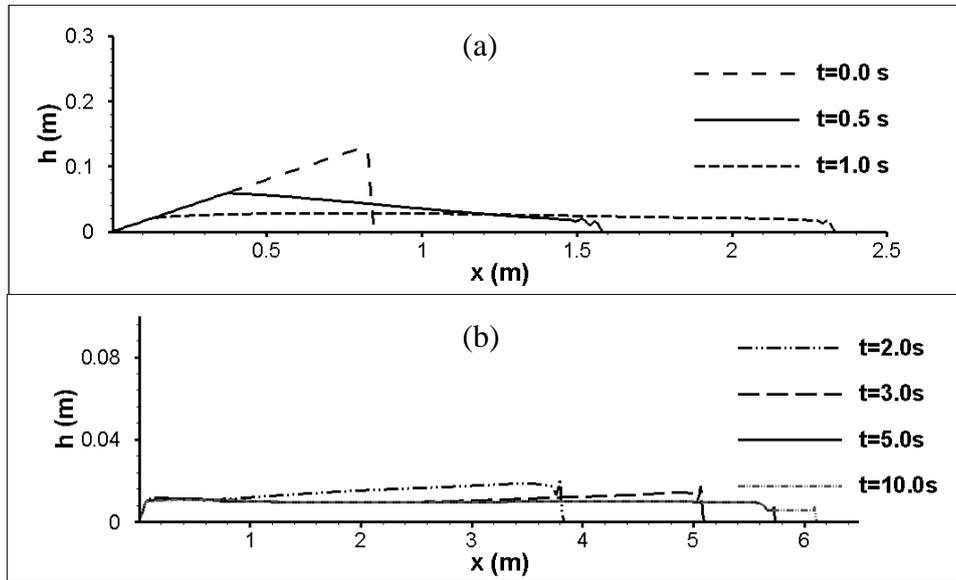


Fig 5. Flow profiles

3. RESULTS

The results of the numerical model are shown in Table 1 in terms of the comparison of arrival time t_a , peak depth y_p and residual depth y_d at three gauges. The overall temporal variation of flow depth at three gauges are shown in Figure 4. In general, the numerical model could produce the flow characteristics of the water-clay mixture in terms of the reproduction of flow peak, propagation speed and residual depth. When flow comes to a stoppage, the profile calculated by the numerical model is the approximate solution of the flow depth under the equilibrium of pressure and the gravitational term and bottom shear stress term in Eq. (2). This profile is depicted by the flow profile at $t = 10.0s$ in Figure 5(b) where the flow is approaching stoppage. At this stage, the bottom shear stress is equal to the yield stress. Therefore for a given set of rheological parameters and bed slope, the residual depth is constant as shown by the numerical result in Figure 4. However, in reality variation of yield stress might exist and therefore the residual depth might differ from the theoretical value. This explains the lower residual depth at Gauge 1 observed in the experiment, as shown in Figure 4.

4. CONCLUSION

In this preliminary work, the wall and bottom shear stress based on the Herschel-Bulkley constitutive relation is adopted in the one-dimensional numerical model. The accuracy and stability of the numerical model are increased with numerical scheme and method and the performance of the model in simulating the release of fine suspension as mudflow is evaluated. The numerical model performed satisfactorily in terms the reproduction of the flow profile and propagation speed of the flow. This preliminary study has provided a comprehensive understanding on the constitutive relation for mudflow simulation and has prepared a basic numerical frame work for the expansion into a two-dimensional flow model.

References

[1] Coussot. P., (1994): Steady, laminar flow of concentrated mud suspensions in open channel, *J. Hydraul. Res.*, 32(4), [pp.535-559](#).

[2] Coussot. P. and Piao. J. M., (1994): On the behavior of fine mud suspensions, *Rheologica Acta.*, 33, [pp.175-184](#).

[3] Govier. G. W. and Aziz. K., (1982): *The flow of complex mixtures in pipes*, Krieger Pub. Co., Melbourne, Florida, United States.

[4] Heinrich. Ph., Boudon. G., Komorowski, J. C., Sparks. R. S. J., Herd. R. and Voight. B., (2001): Numerical simulation of the December 1997 debris avalanche in Montserrat, Lesser Antilles, *Geophy. Research Letters*, 28(13), [pp.2529-2532](#).

[5] Hungr. O., (1995): A model for the runout analysis of rapid flow slides, debris flows, and avalanches, *Canadian Geotech. J.*, 32(4), [pp.610-623](#).

[6] Jain. S. C, (2001): *Open channel flow*, John Wiley and Sons, Inc., New York.

[7] Jeyapalan. J. K., Duncan. J. M. and Seed. H. B., (1983): Analyses of flow failures of mine tailing dams, *J. Geotech. Eng., ASCE*, 109(2), [pp.150-171](#).

[8] Laigle. D., Coussot. P., (1997): Numerical modeling of mudflows, *J. Hydraul. Eng., ASCE*, 123, [pp.617-623](#).

[9] O'Brien. J. S. and Julien. P. Y., (1988): Laboratory analysis of mudflow properties, *J. Hydraul. Eng., ASCE*, 114, [pp.877-887](#).

[10] O'Brien. J. S., Julien. P. Y., and Fullerton. W. T., (1993): Two-dimensional water flood and mudflow simulation, *J. Hydraul. Eng., ASCE*, 119, [pp.244-261](#).

[11] Rickenmann. D., Laigle. D., McArdeil. B. W. and Hübl. J., (2006): Comparison of 2D debris-flow simulation models with field events, *Comp. Geosc.*, 10, [pp.241-264](#).

[12] Schamber. D. R. and MacArthur. R. C., (1985): One-dimensional model for mud flows, *Hydraul. And Hydrol. in the Small Comp. Age*, ASCE, 1 [pp.1334-1339](#).

[13] Tannehill. J. C., Anderson. D. A. and Pletcher. R. H., (1997): *Computational fluid mechanics and heat transfer*, Taylor and Francis, United States.

[14] Zanuttigh. B., Lamberti. A., (2004): Analysis of debris wave development with one-dimensional shallow-water equations, *J. Hydraul. Eng., ASCE*, 130, [pp.293-304](#).